The halo model and cosmological power spectra

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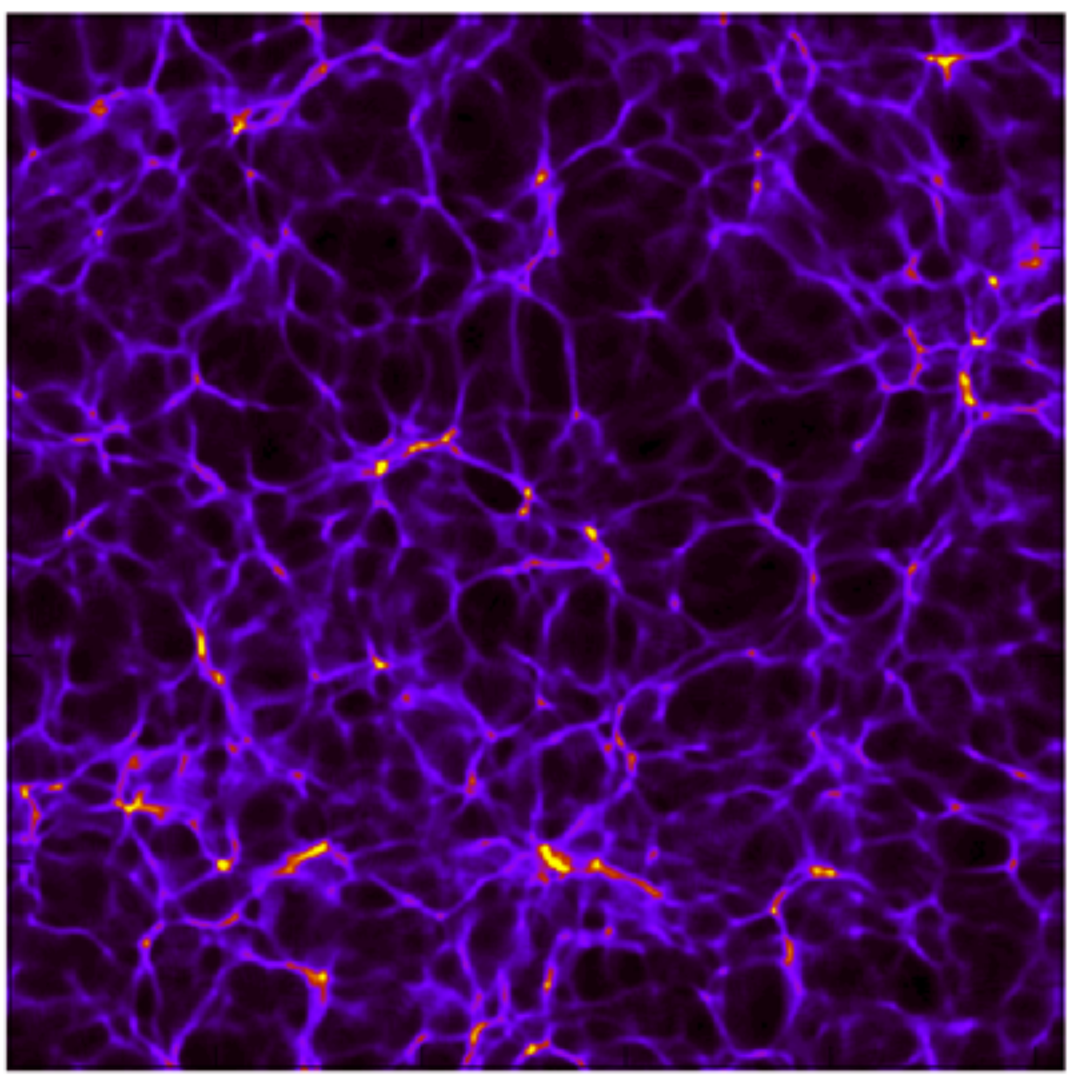

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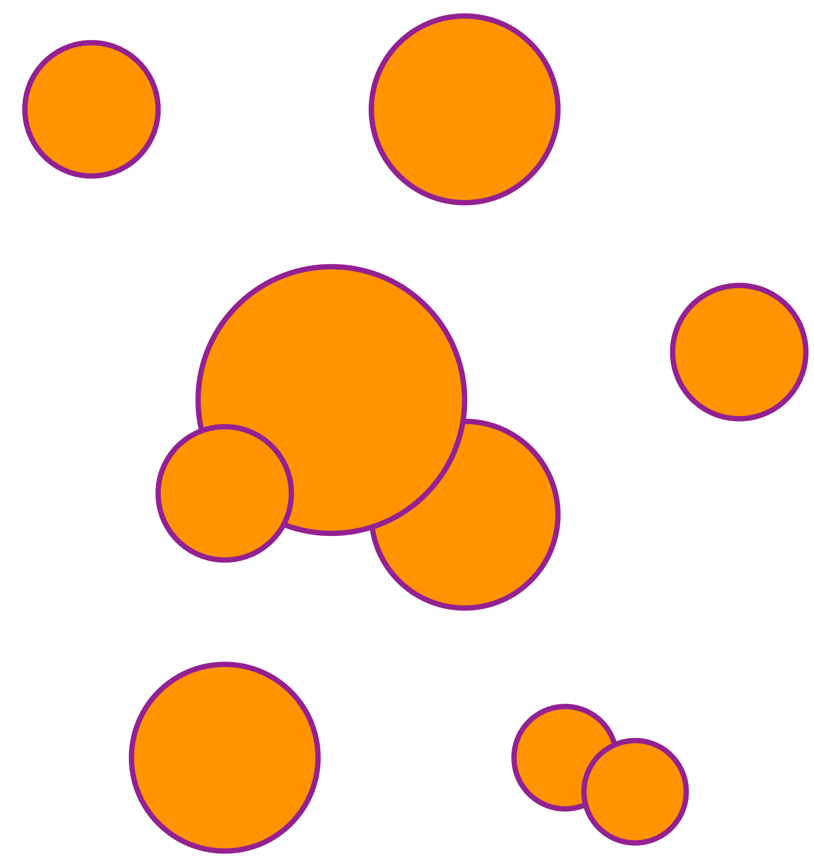
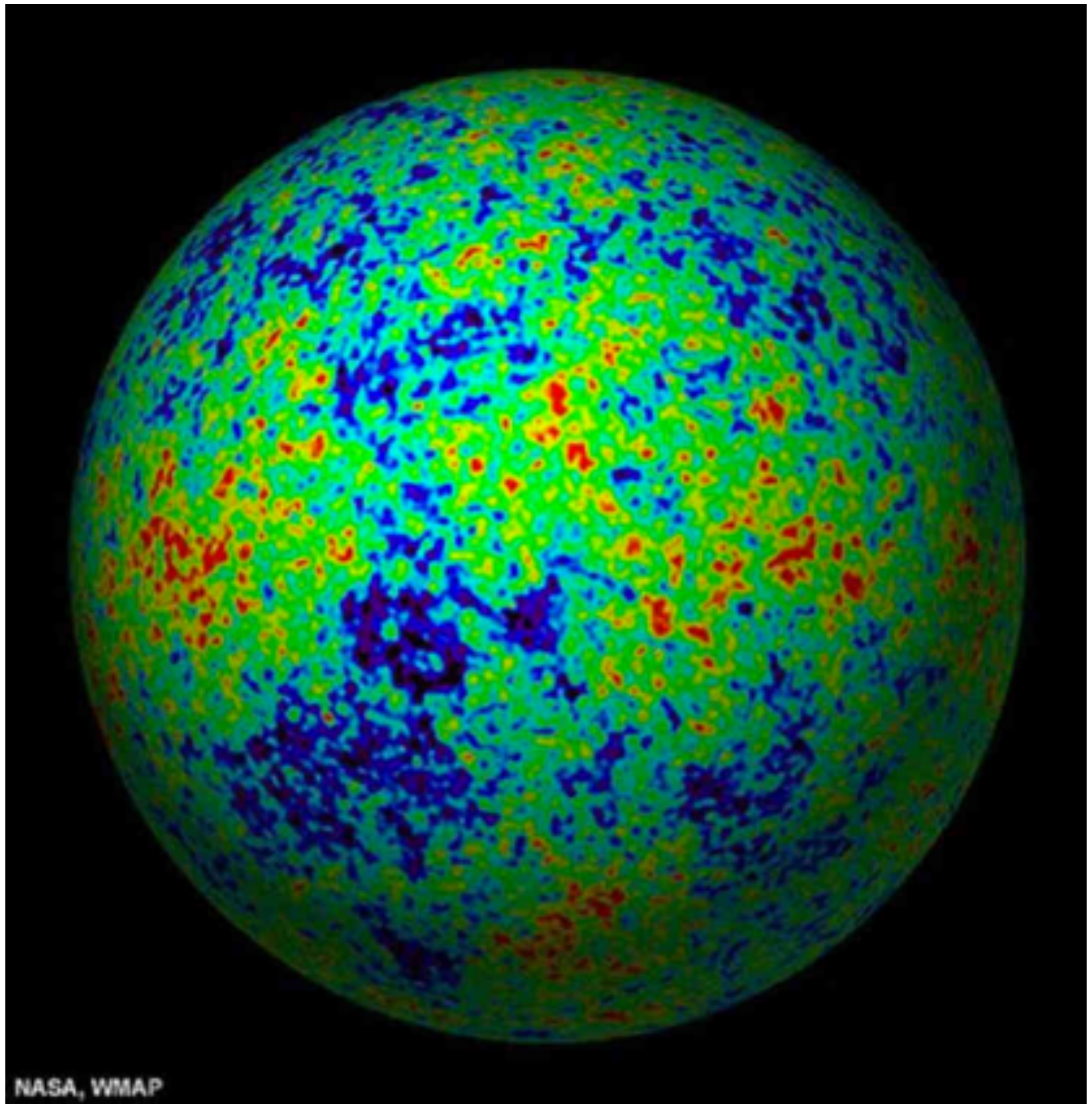


Introduction

- Aim to understand the distribution of large-scale structure (matter) in the late ($z < 2$) Universe.
- Structure is traced out by galaxies, which we can see, but consists of a dark matter exoskeleton, often called the 'cosmic web'
- This structure can be simulated easily, and measured directly via weak gravitational lensing
- Different cosmological models predict very different patterns for the cosmic web, and can thus be distinguished using weak lensing.

The problem

- Structure in the late Universe is non-linear and complex
- Initial perturbations are linear and Gaussian
- Gravitational collapse makes things non-linear and non-Gaussian
- Late Universe must be modelled (e.g., weak lensing)
- Necessary to do this as a function of model (DE, MG, massive neutrinos)



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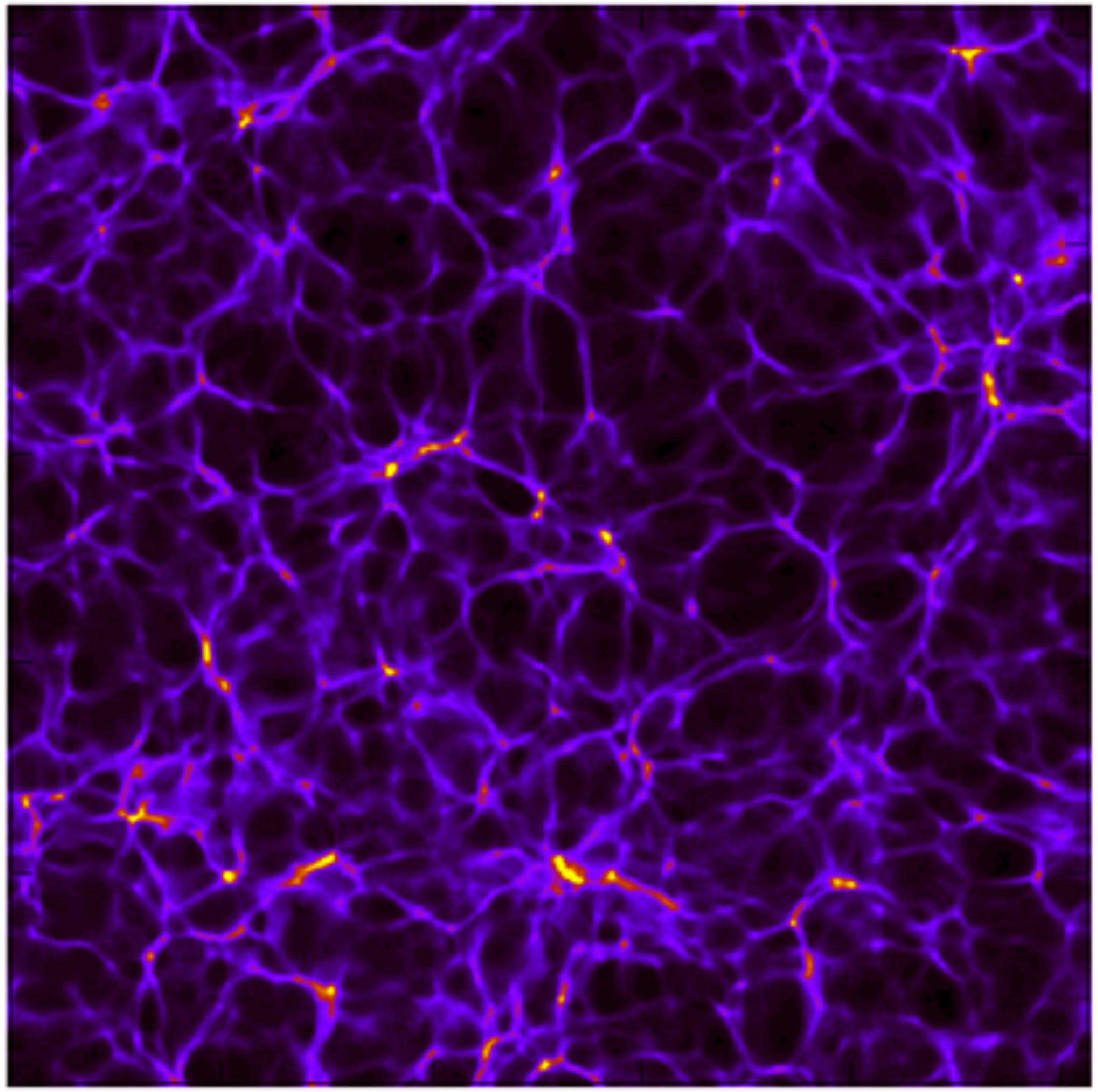

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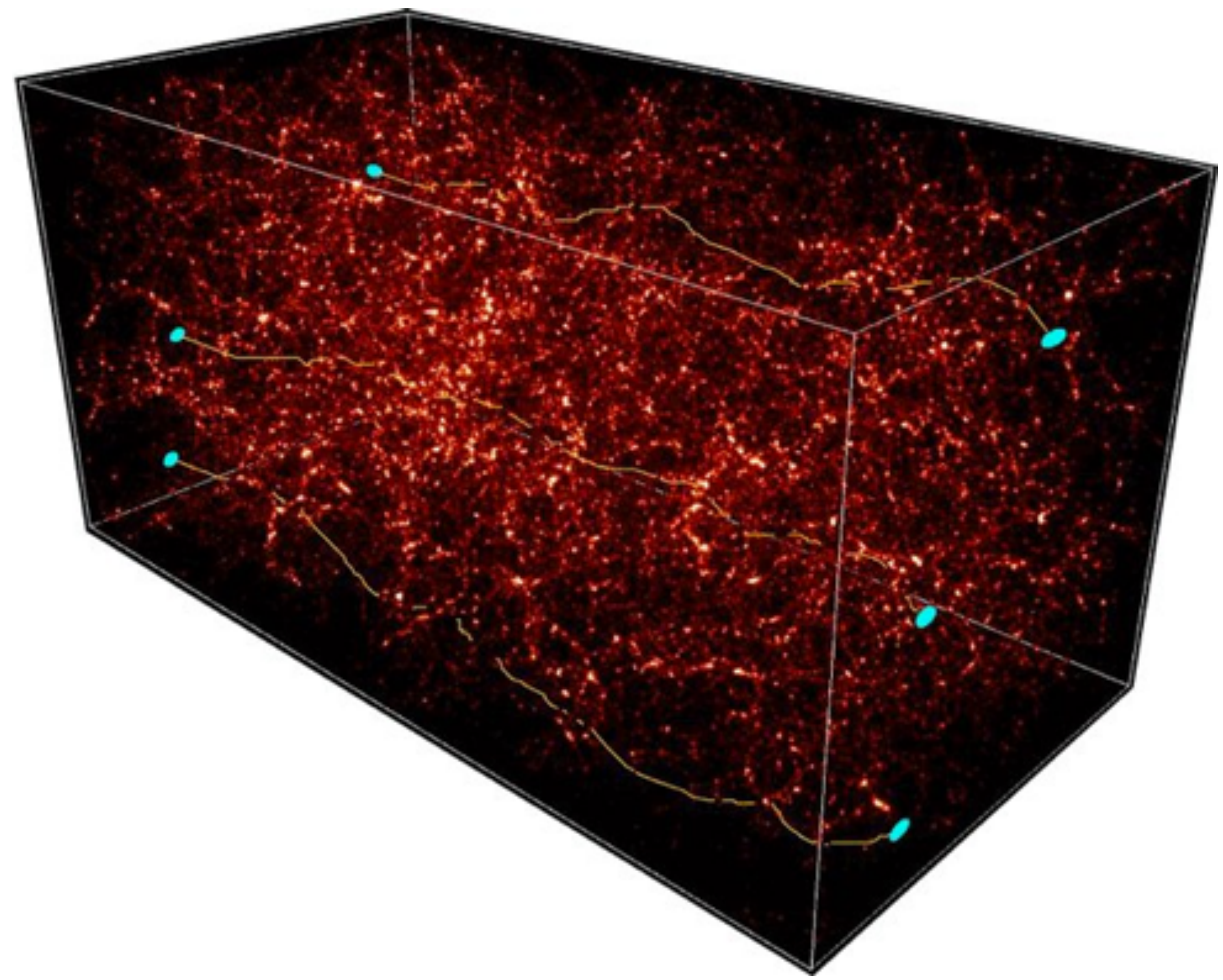
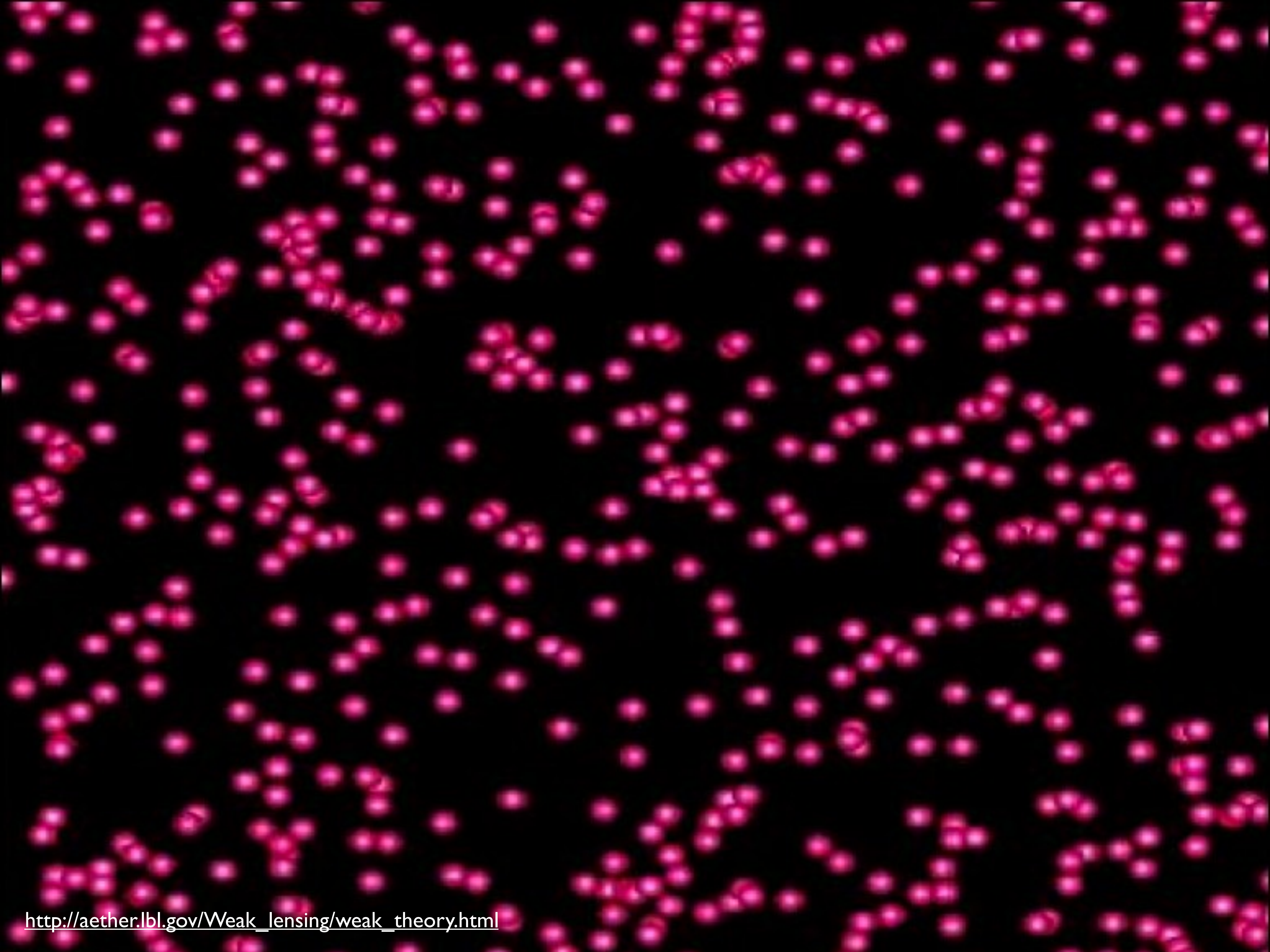
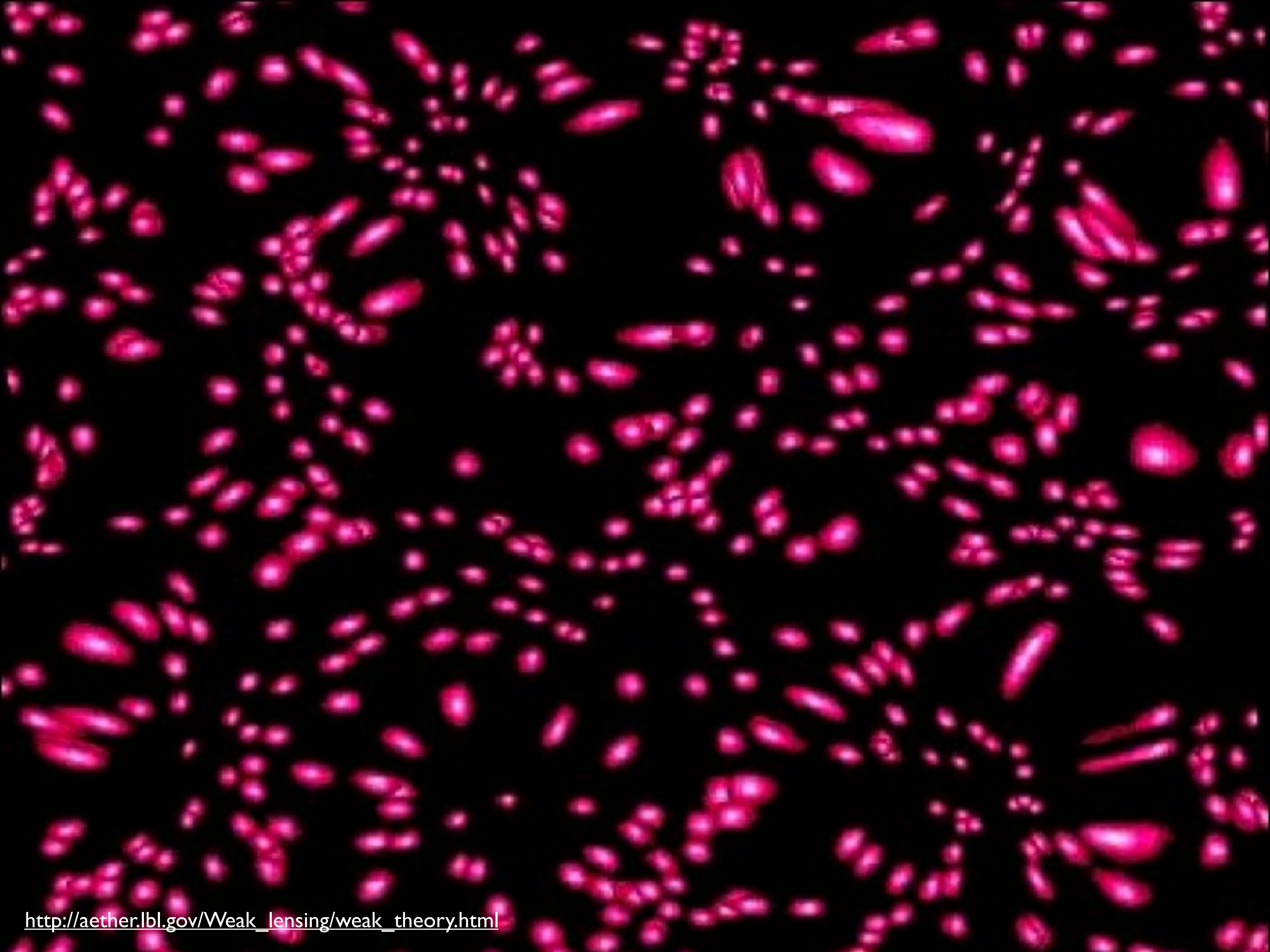




http://aether.lbl.gov/Weak_lensing/weak_theory.html

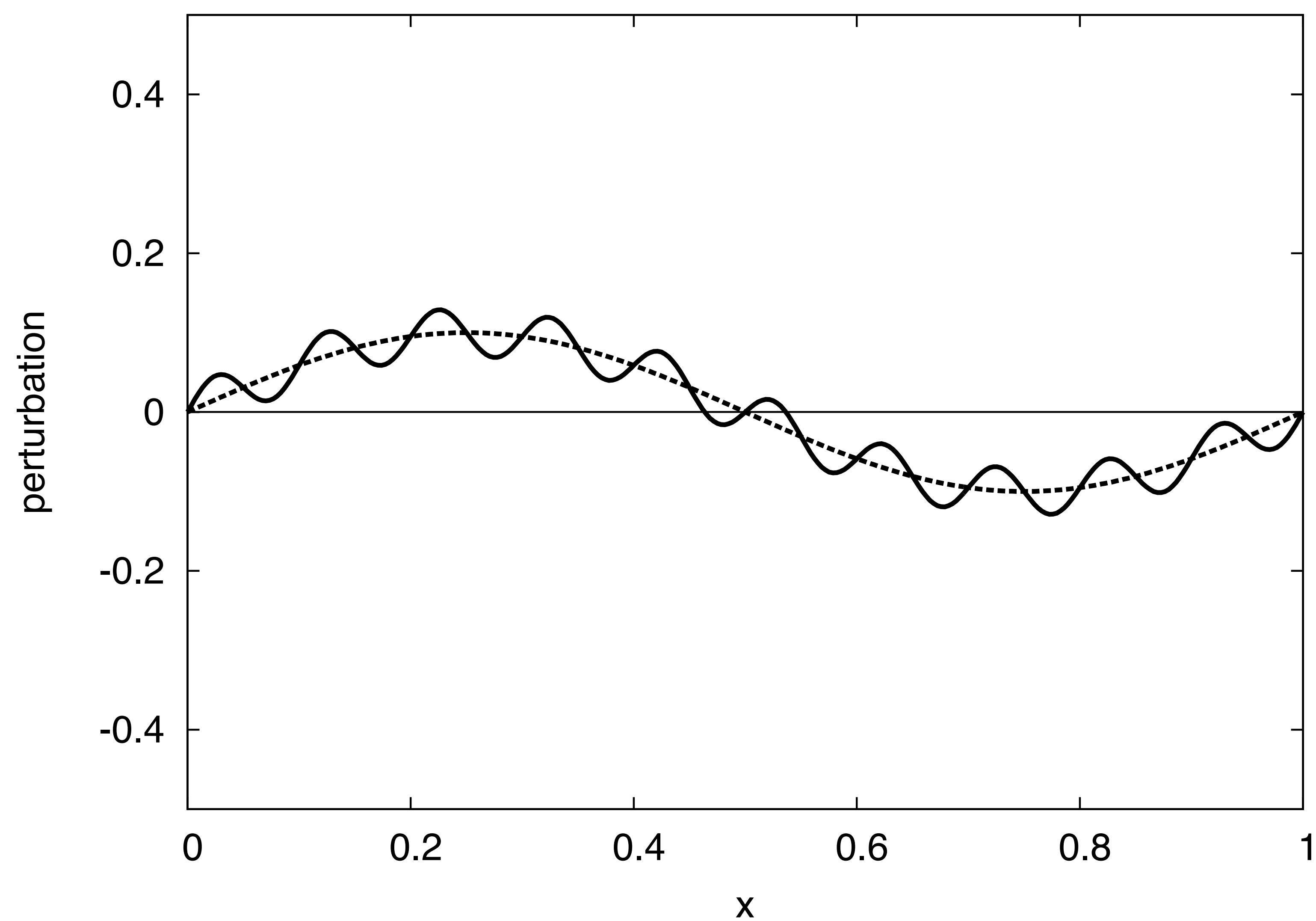


http://aether.lbl.gov/Weak_lensing/weak_theory.html

Linear perturbation theory

$$1 + \delta(\mathbf{x}) = \frac{\rho(\mathbf{x})}{\bar{\rho}}$$

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G\bar{\rho}_{\rm m}\delta$$

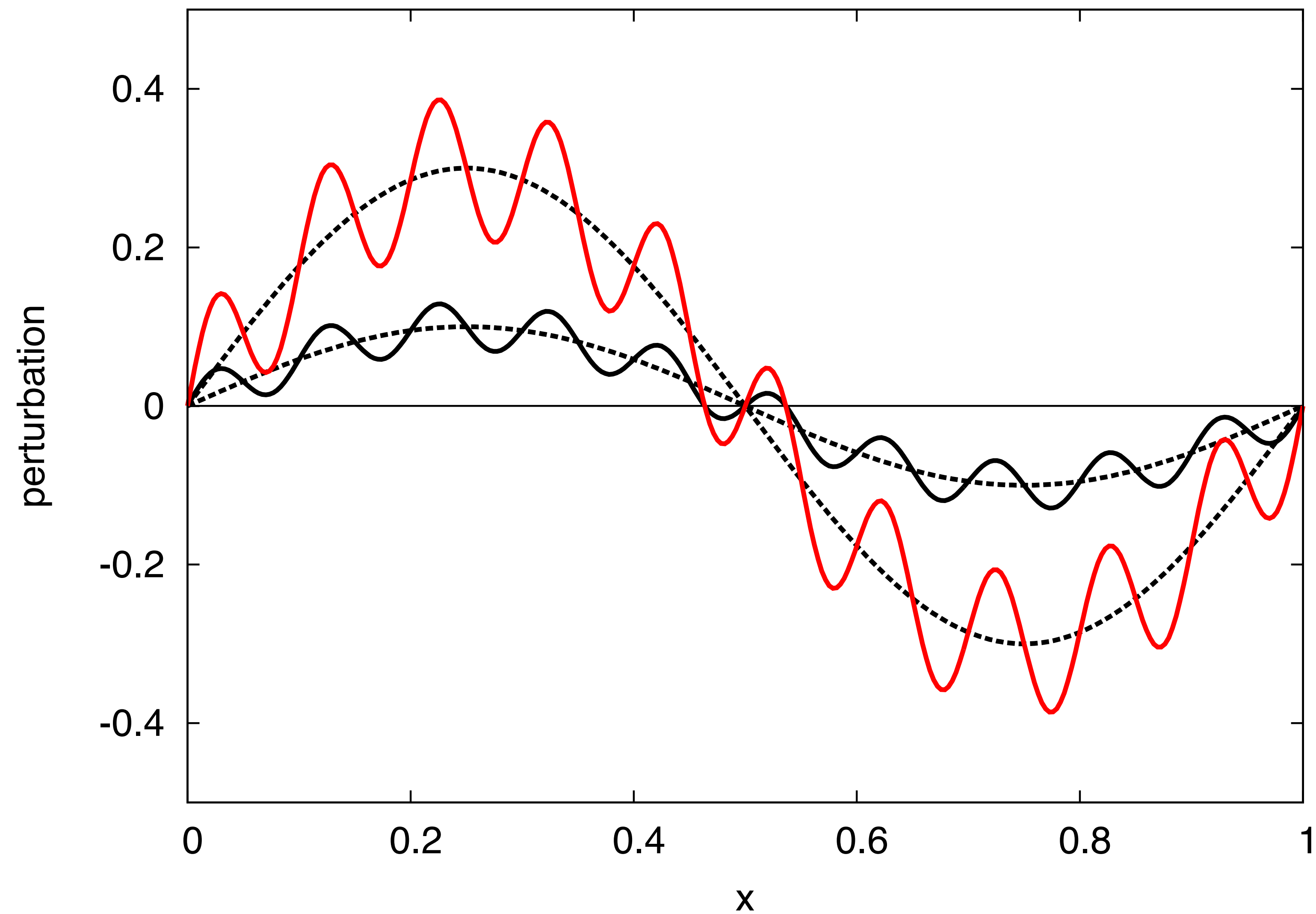


Linear perturbation theory

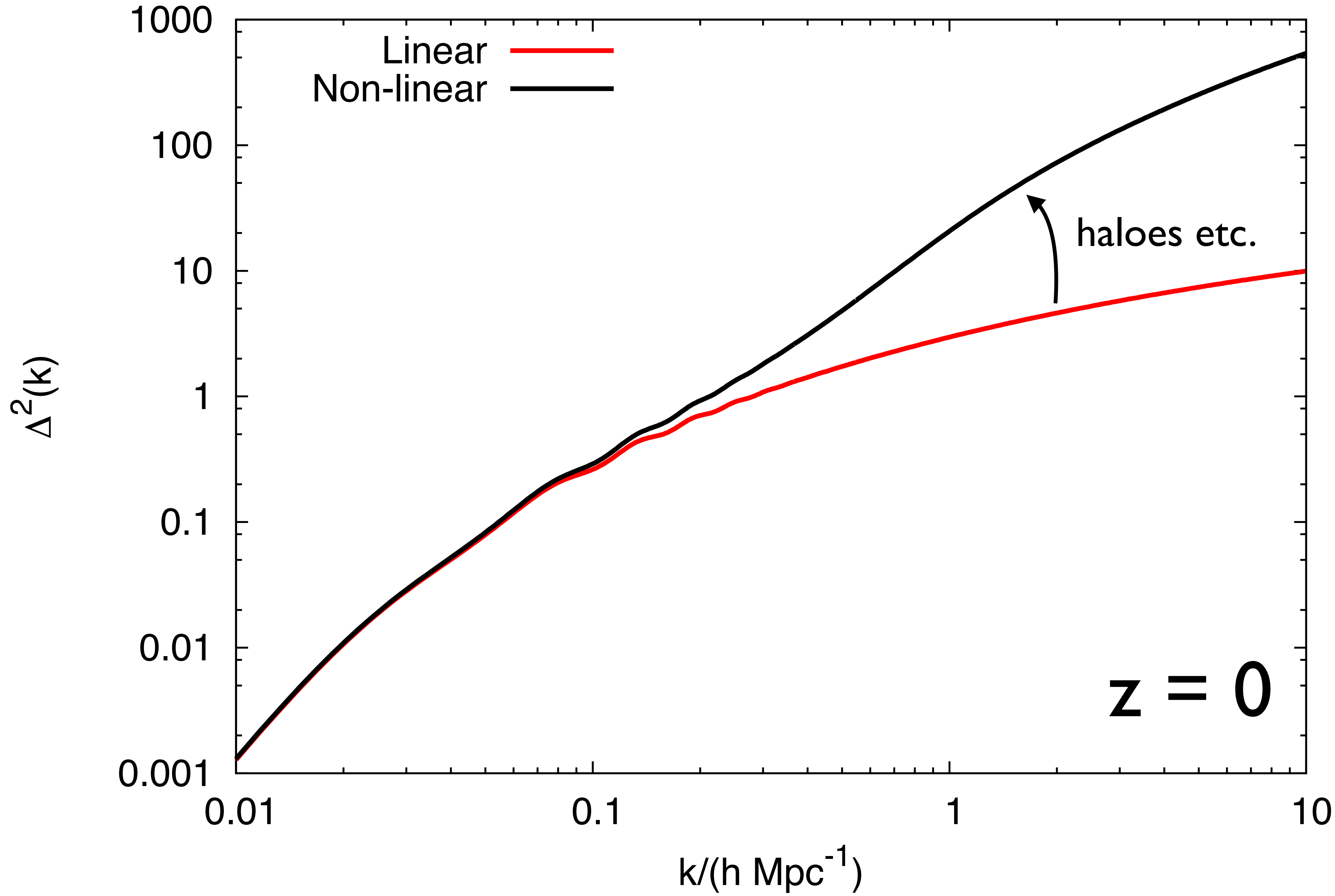
$$1 + \delta(\mathbf{x}) = \frac{\rho(\mathbf{x})}{\bar{\rho}}$$

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G \bar{\rho}_{\mathrm{m}} \delta$$

$$\delta \propto a$$



The power spectrum of matter fluctuations

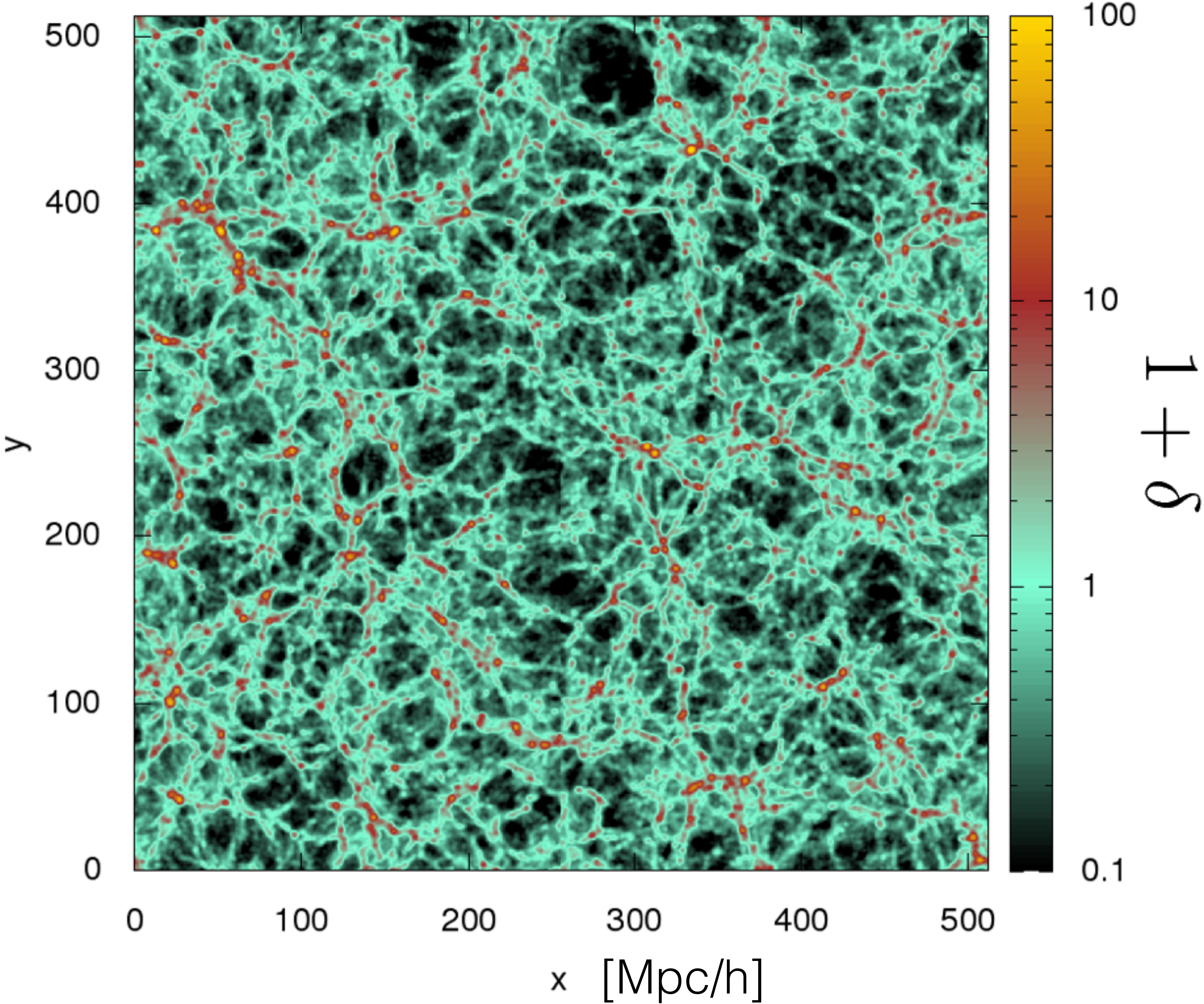


$$\Delta^2(k,z) = 4\pi V \left(\frac{k}{2\pi}\right)^3 P(k,z) \qquad P(k,z) = \langle |\delta_k|^2 \rangle$$

N-body simulations

- Represent the smooth density field with '*N*' particles
  - 1. Calculate the gravitational forces
  - 2. Move the particles
  - 3. Repeat
- Periodic boundaries
- Expanding box

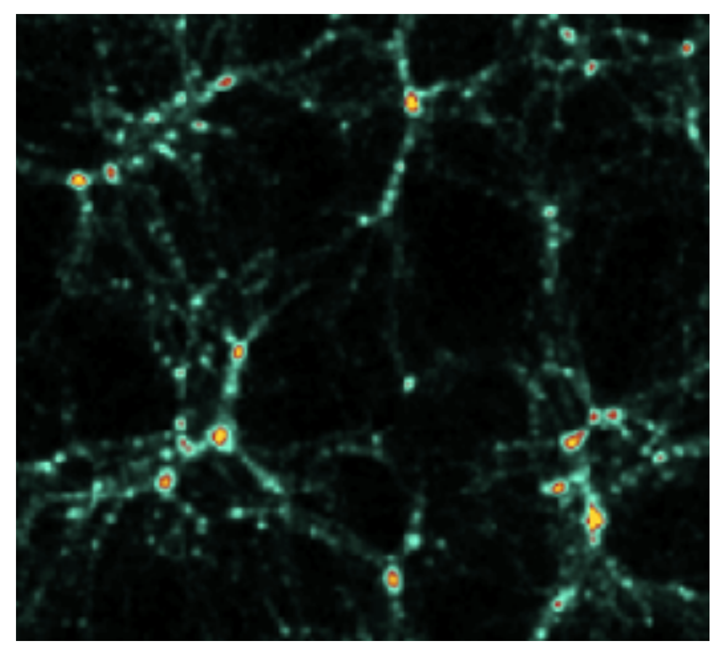
$$\ddot{\mathbf{r}} + 2H\dot{\mathbf{r}} = -\frac{1}{a^2}\nabla\Phi$$

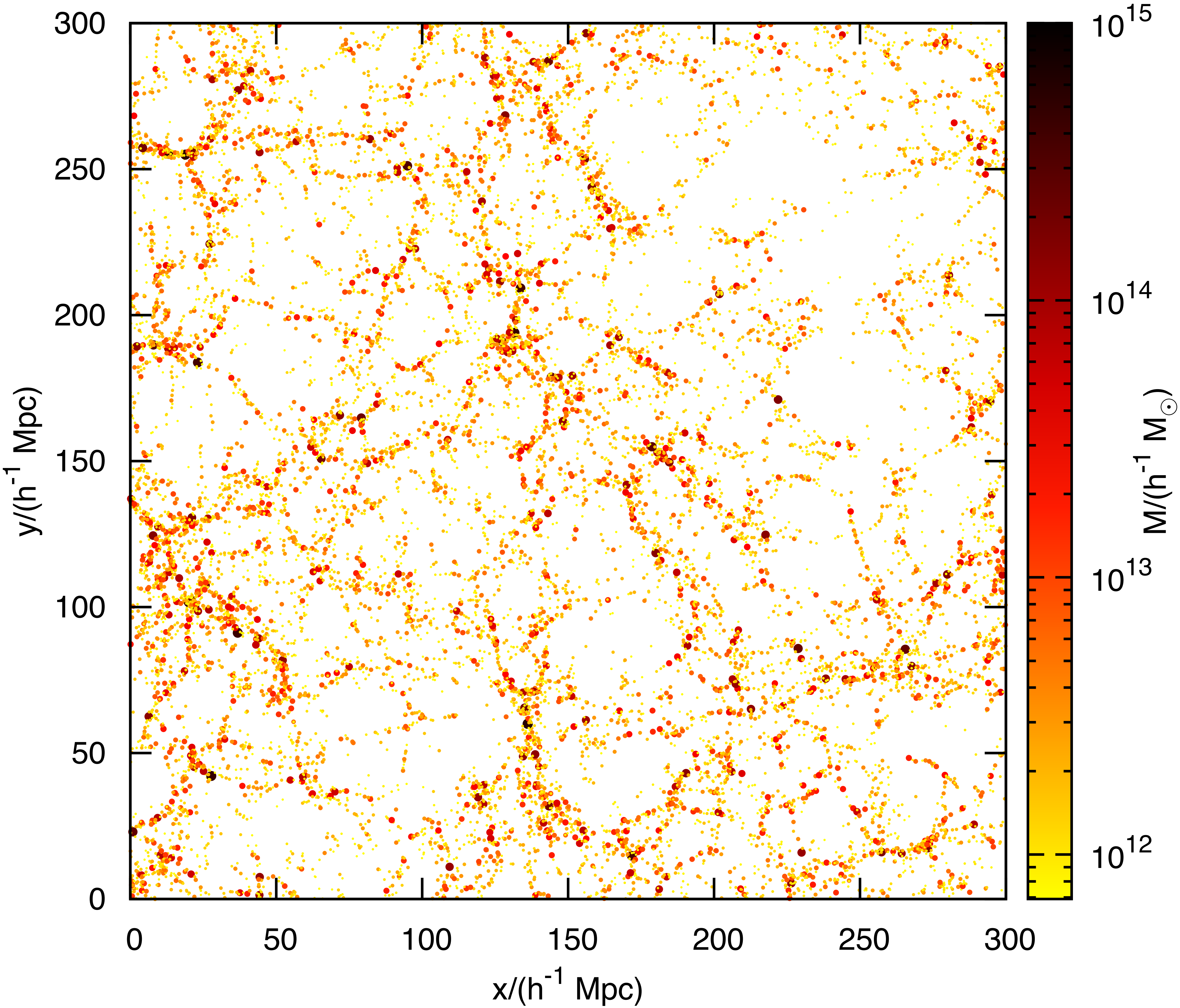


500
400
300
200
100
0
y
0
100
200
300
400
500
x [Mpc/h]
100
10
1
0.1
$1+\delta$

Simulation limitations

- Solve gravity only problem to high accuracy
- Expensive in terms of computing power
- It will **never** be possible to run accurate simulations for every model under consideration
- Investigating cosmological parameter space:
  - ΛCDM
  - Dark Energy (interacting?)
  - Massive neutrinos
  - Dark matter types
  - Modified gravity
  - Baryons?





300
250
200
150
100
50
0
0
50
100
150
200
250
300
y/(h-1 Mpc)
x/(h-1 Mpc)
10^15
10^14
10^13
10^12
M/(h-1 M⊙)

The halo model

- Can be used to predict the clustering of matter (as well as of haloes or of galaxies) in the Universe
- Distinct from HALOFIT (Smith et al. 2003)

large-scales ~ linear

$$\Delta^2(k) = \Delta^2_{2\mathrm{H}} + \Delta^2_{1\mathrm{H}}$$

small scales - contribution from haloes

$$\Delta^2_{1\mathrm{H}}(k) = 4\pi \left(\frac{k}{2\pi}\right)^3 \frac{1}{\bar{\rho}^2} \int_0^\infty M^2 W^2(k, M) f(M)\, \mathrm{d}M$$

$$\Delta^2_{1\mathrm{H}}(k) = 4\pi \left(\frac{k}{2\pi}\right)^3 \frac{1}{\bar{\rho}^2} \int_0^\infty M^2 W^2(k,M) f(M)\,\mathrm{d}M$$

Ingredients

- Halo mass function

$$f(\nu) = A\left[1 + \frac{1}{(a\nu^2)^p}\right] \mathrm{e}^{-a\nu^2/2}$$

- Peak to mass relation

$$\nu = \frac{\delta_\mathrm{c}}{\sigma(M)}$$

- Halo density profiles

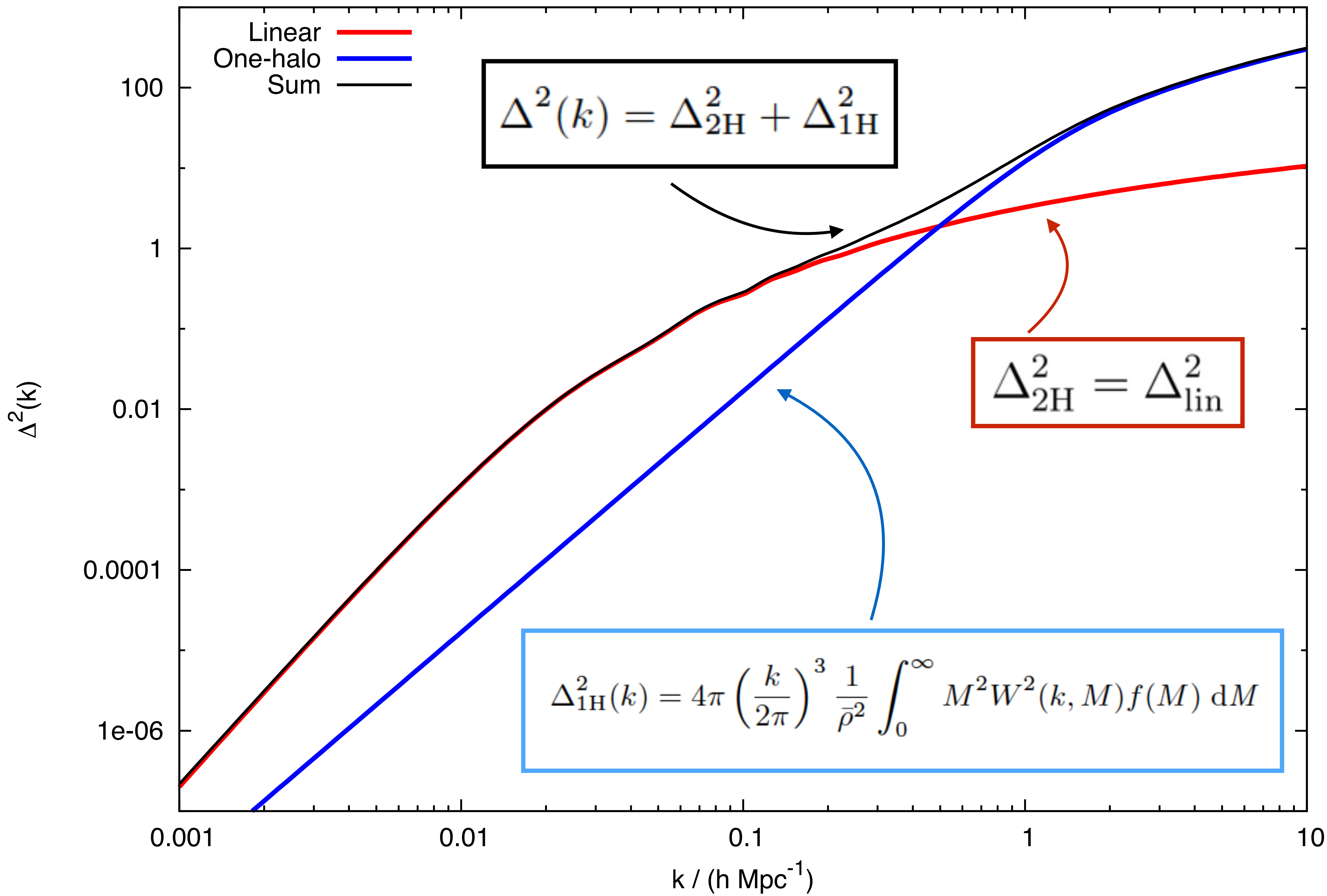
$$\rho(r) = \frac{\rho_\mathrm{N}}{(r/r_\mathrm{s})(1+r/r_\mathrm{s})^2}$$

- Halo radius

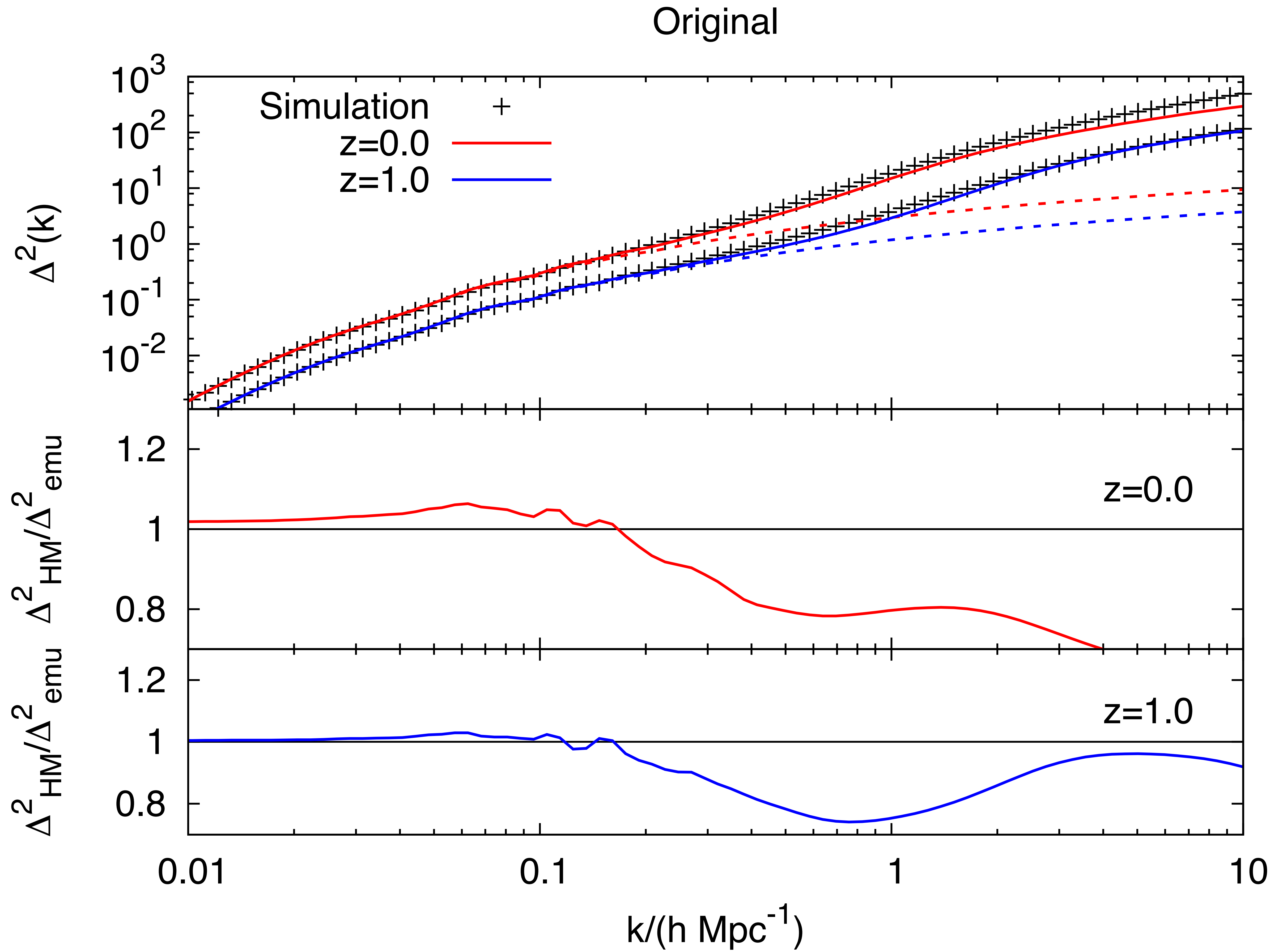
$$r_\mathrm{v} = \left(\frac{3M}{4\pi\Delta_\mathrm{v}\bar{\rho}}\right)^{1/3}$$

- Halo concentration

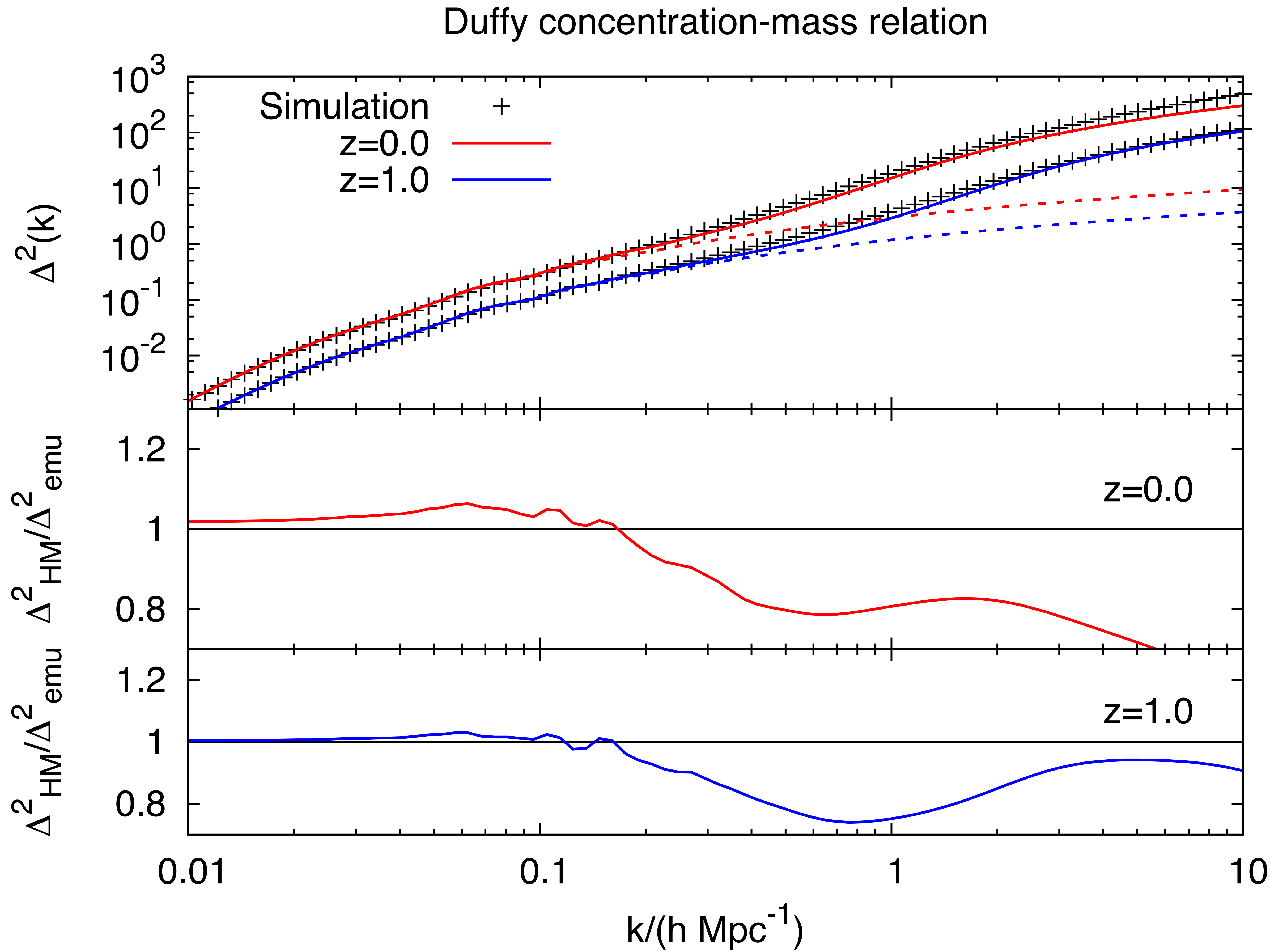
$$c(M,z) = A\frac{1+z_\mathrm{f}}{1+z}$$



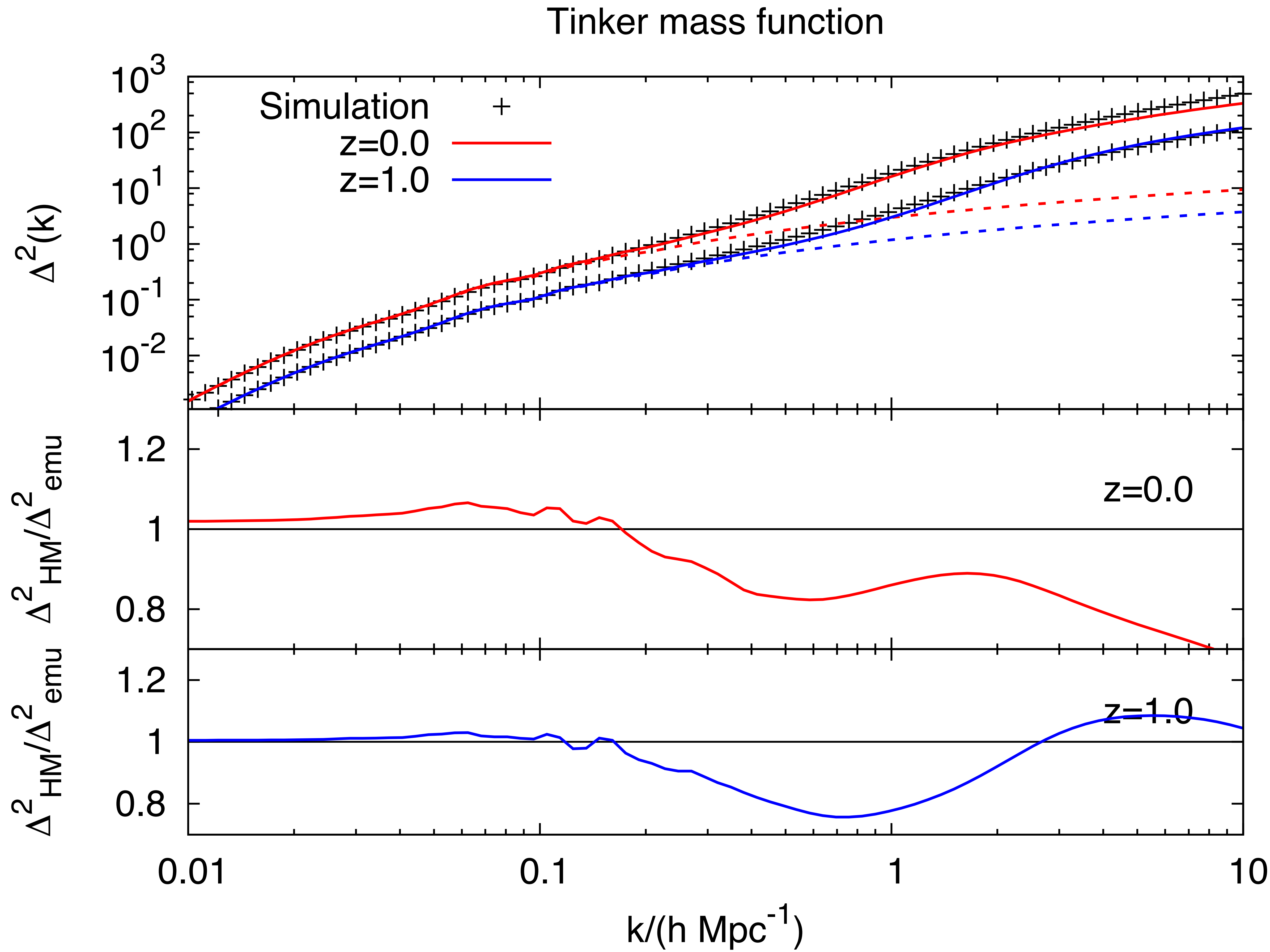
Linear
One-halo
Sum
$\Delta^2(k) = \Delta^2_{2H} + \Delta^2_{1H}$
$\Delta^2_{2H} = \Delta^2_{lin}$
$\Delta^2_{1H}(k) = 4\pi \left(\frac{k}{2\pi}\right)^3 \frac{1}{\bar{\rho}^2} \int_0^\infty M^2 W^2(k, M) f(M)\, dM$
100
1
0.01
0.0001
1e-06
0.001
0.01
0.1
1
10
$\Delta^2(k)$
k / (h Mpc^{-1})



Original
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
$\Delta^2(k)$
Simulation
z=0.0
z=1.0
1.2
1
0.8
$\Delta^2_{HM}/\Delta^2_{emu}$
z=0.0
1.2
1
0.8
$\Delta^2_{HM}/\Delta^2_{emu}$
z=1.0
0.01
0.1
1
10
$k/(h\ Mpc^{-1})$

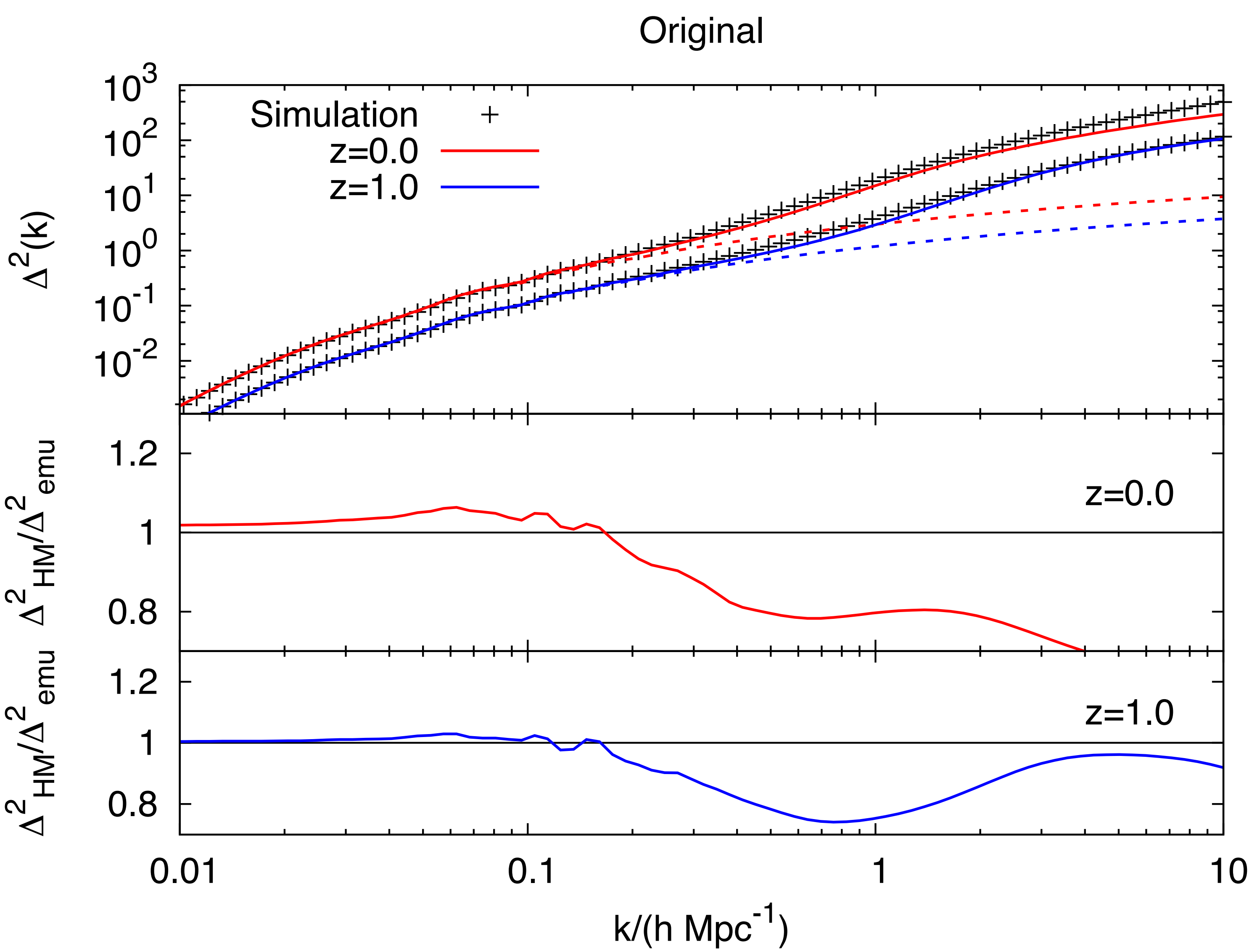


Duffy concentration-mass relation
Simulation
z=0.0
z=1.0
$\Delta^2(k)$
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
$\Delta^2_{HM}/\Delta^2_{emu}$
1.2
1
0.8
z=0.0
z=1.0
0.01
0.1
1
10
k/(h Mpc^{-1})



Tinker mass function
Simulation
z=0.0
z=1.0
$\Delta^2(k)$
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
$\Delta^2_{HM}/\Delta^2_{emu}$
1.2
1
0.8
z=0.0
z=1.0
0.01
0.1
1
10
k/(h Mpc^{-1})

Standard halo model

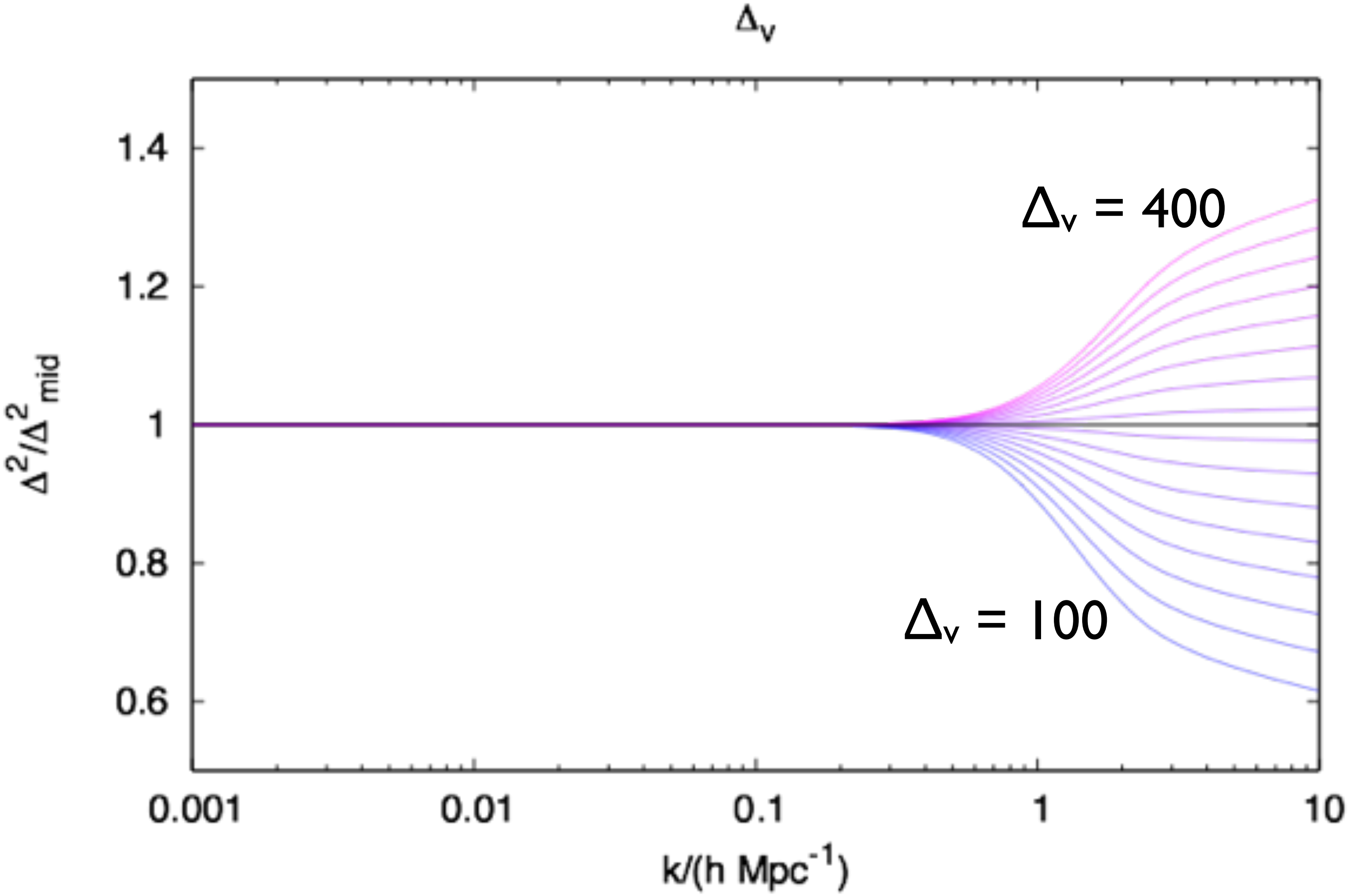


Problems

- Perturbation theory at large scales
- Transition region is problematic (voids, under-densities, NL bias)
- Filamentary structure missing
- Halo asphericity ignored
- Tidal alignment of haloes
- Assumes all objects virialized
- Halo substructure ignored
- Scatter in halo properties at fixed mass ignored

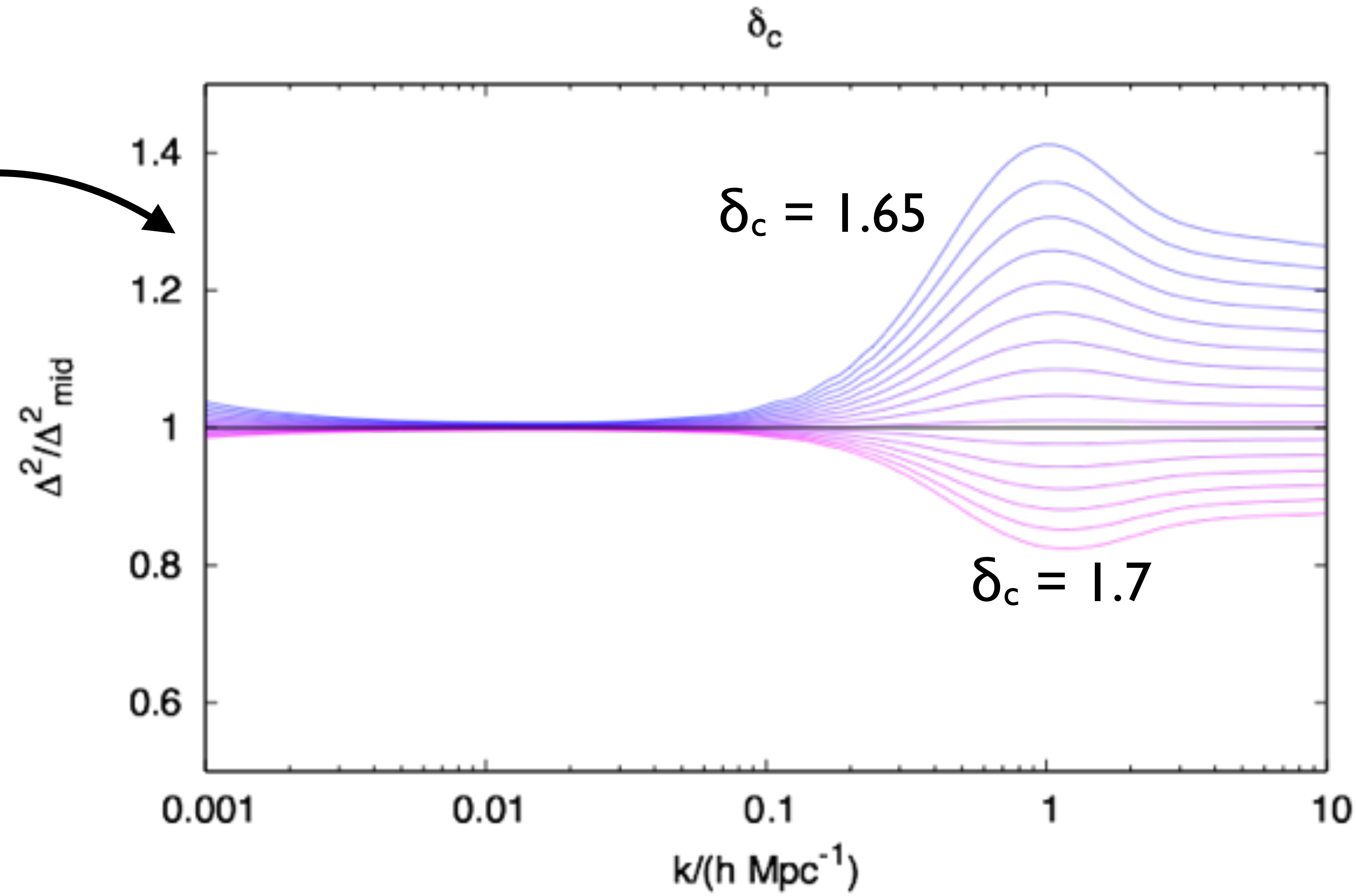
Method

- We want to remedy the inaccuracy of the halo model without breaking it
- Fixing all of the problems whilst being fully consistent would be hard
- Instead opt for the simpler goal of generating 'effective haloes', whose simple halo power spectrum accurately matches that of simulations



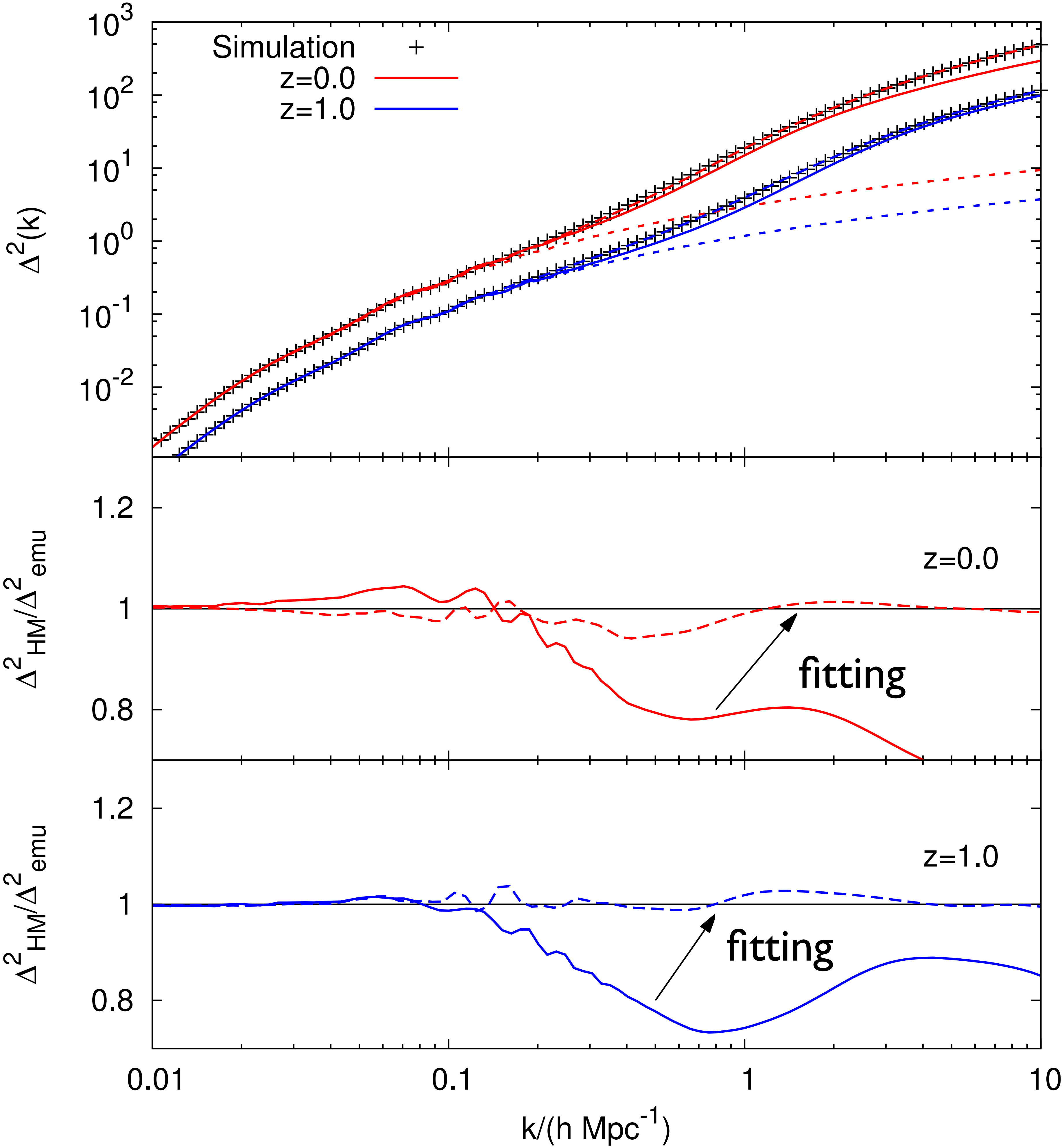
Variations in virial density

$\Delta_v = 200$



Variations in linear collapse density

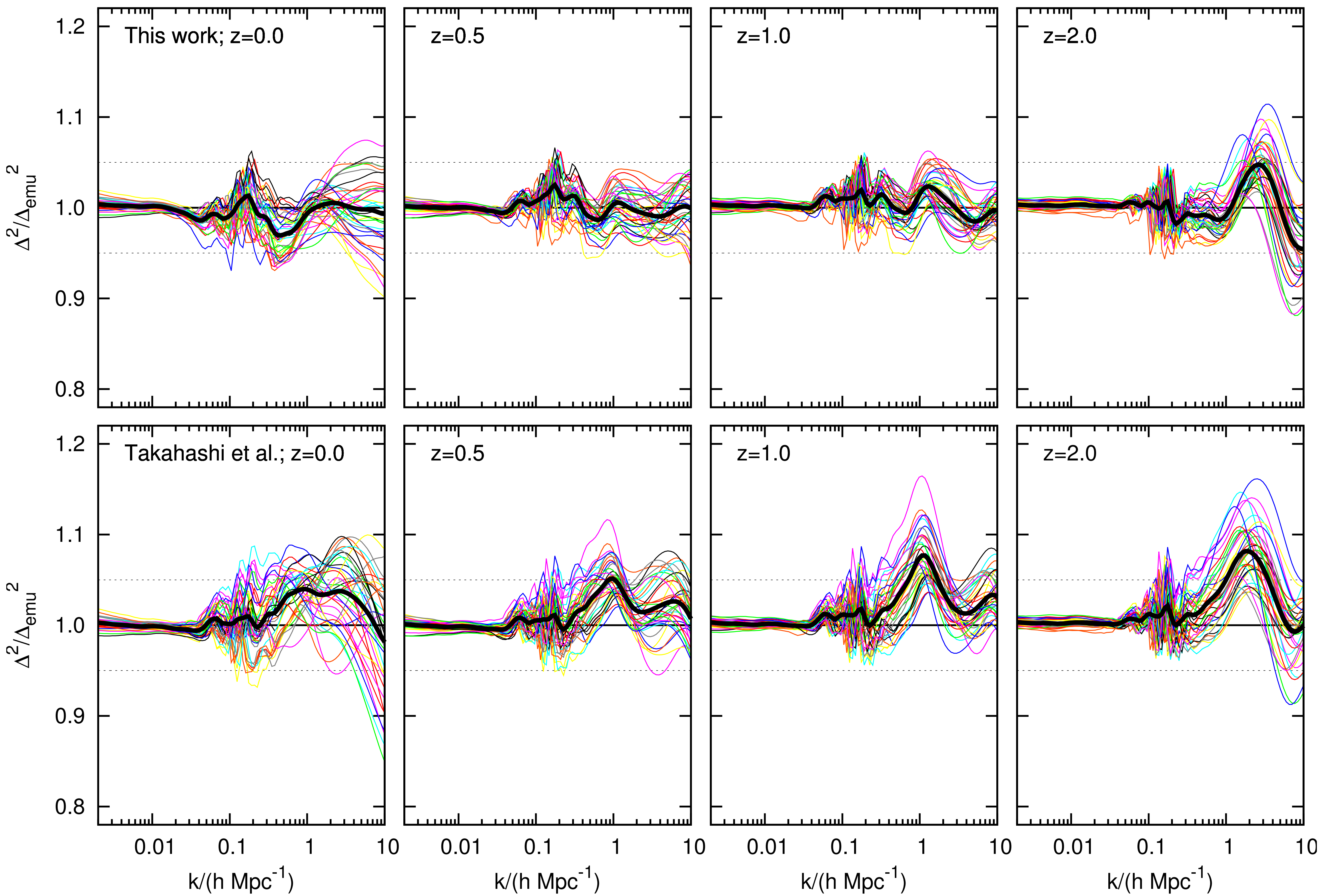
$\delta_c = 1.686$



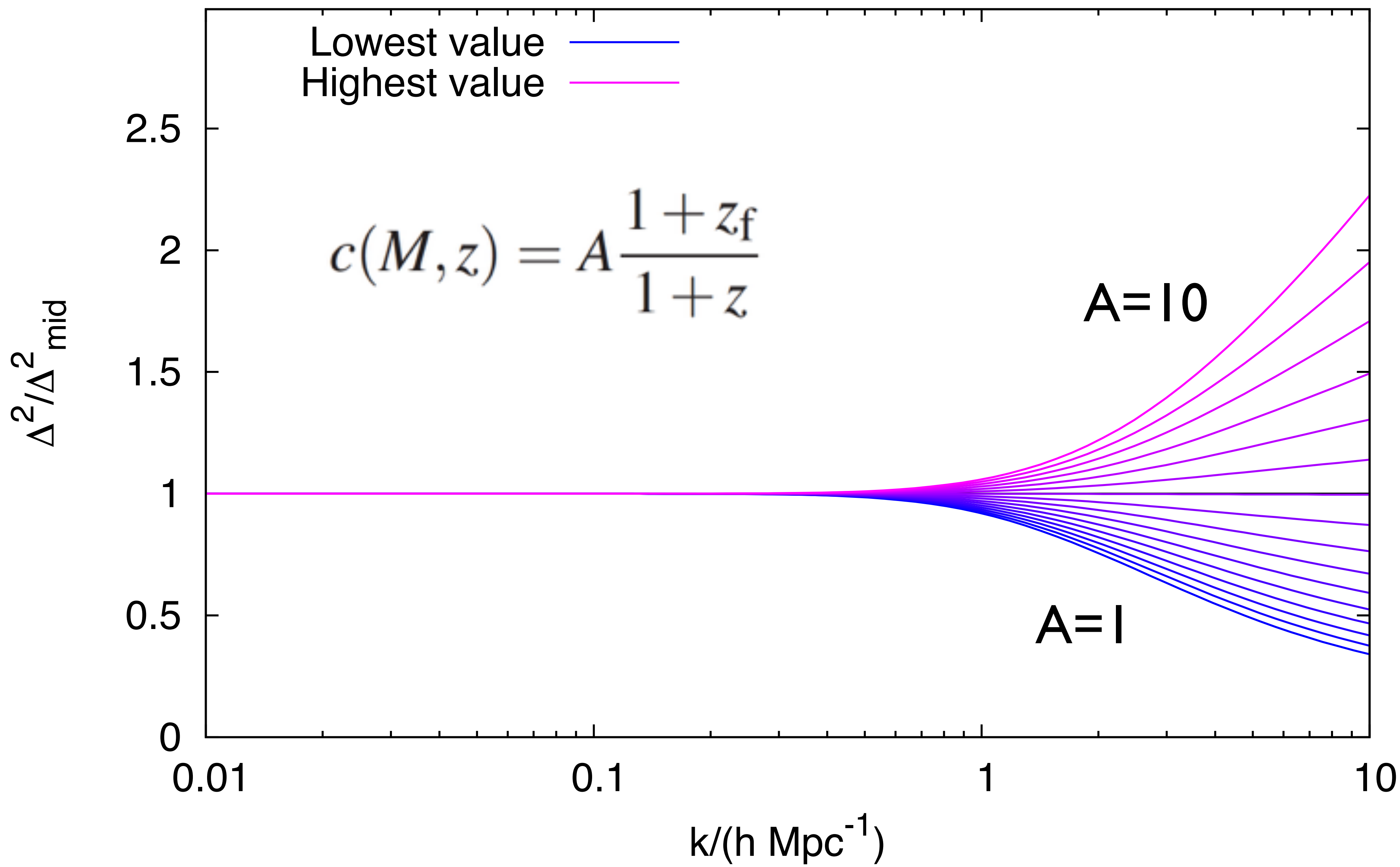
Simulation
z=0.0
z=1.0
$\Delta^2(k)$
$\Delta^2_{HM}/\Delta^2_{emu}$
z=0.0
fitting
z=1.0
fitting
$k/(h\ Mpc^{-1})$

Cosmic Emu (simulations) fit and comparison

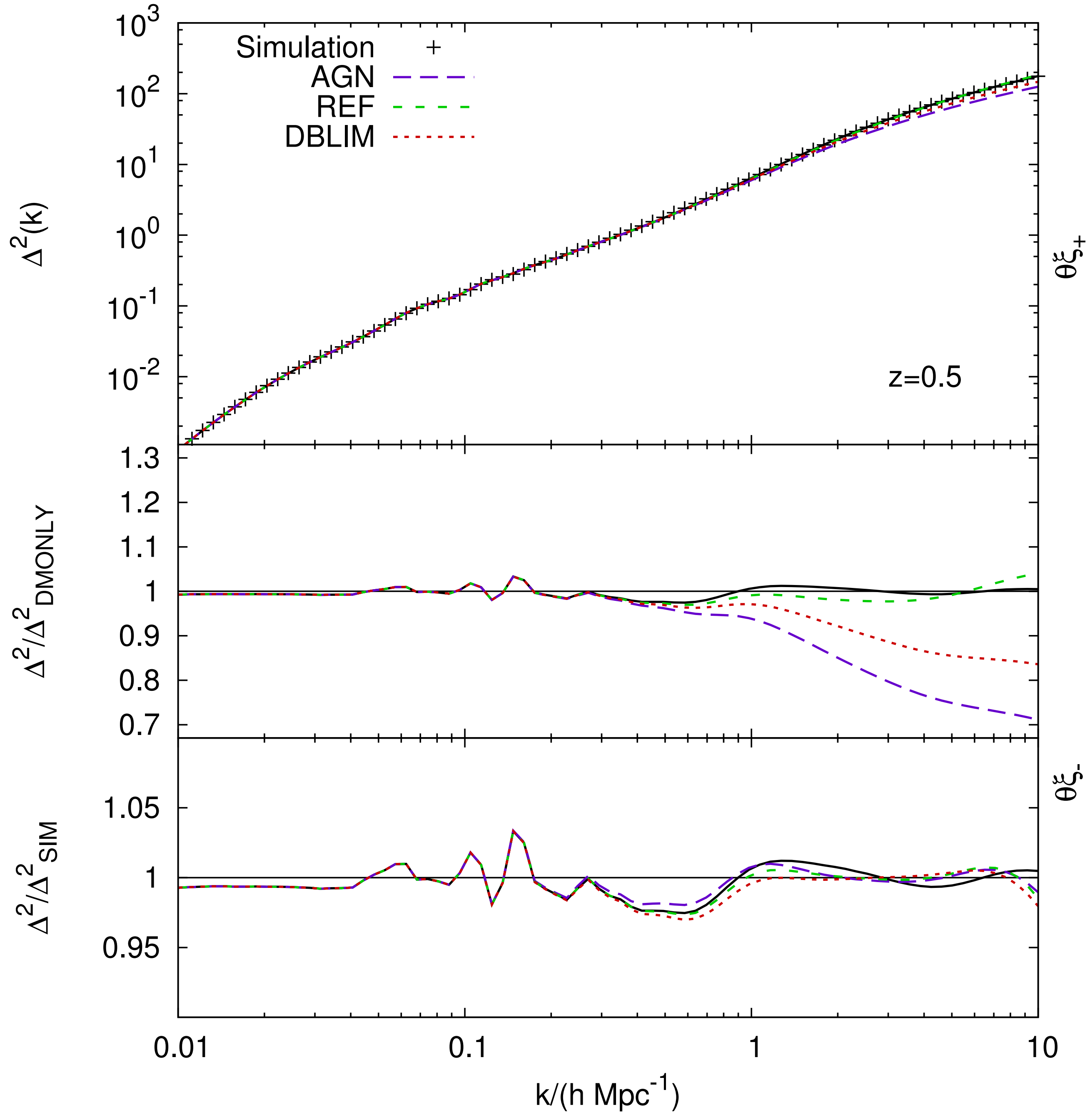
(Heitmann 2014)

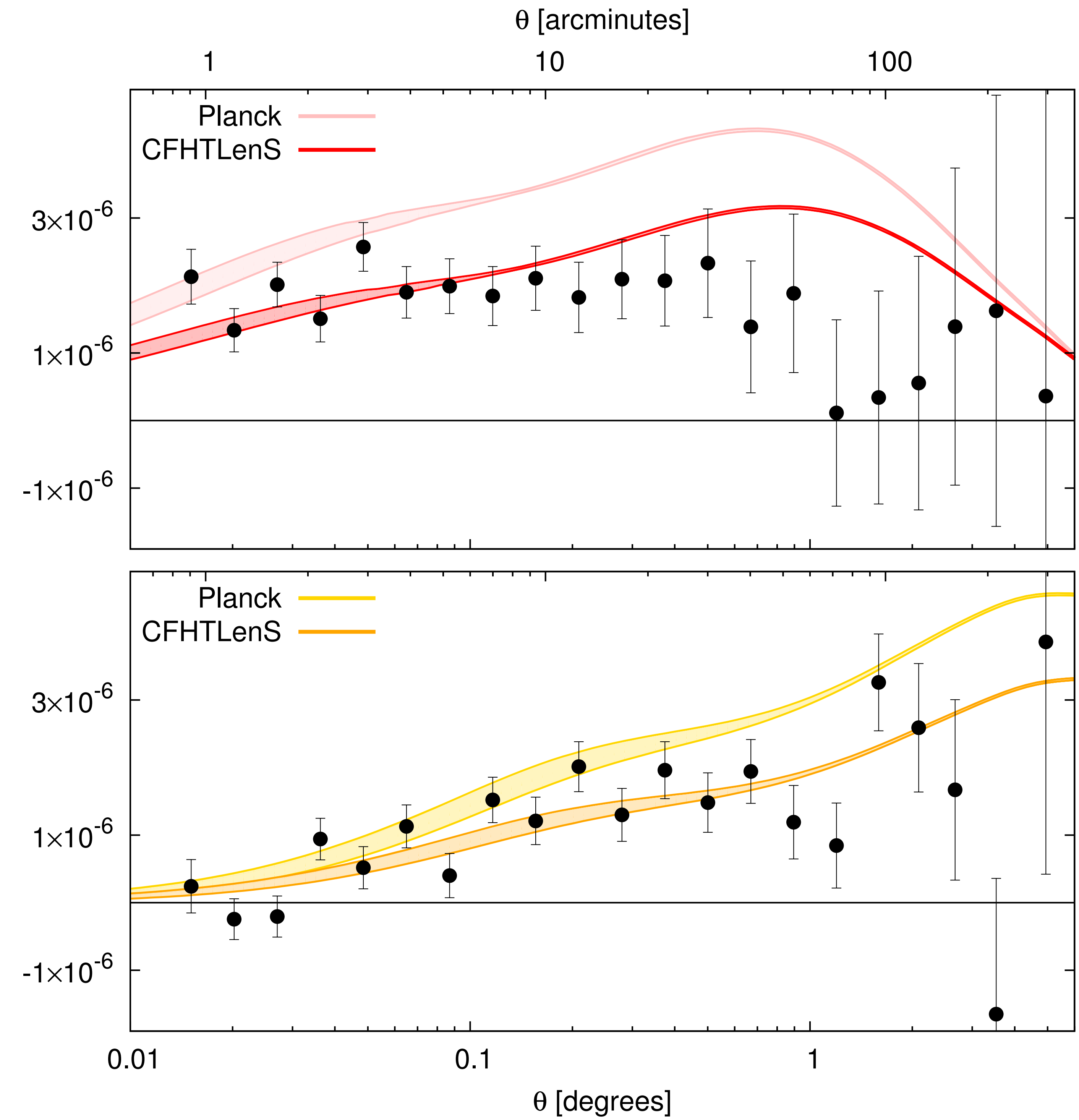


Baryonic feedback

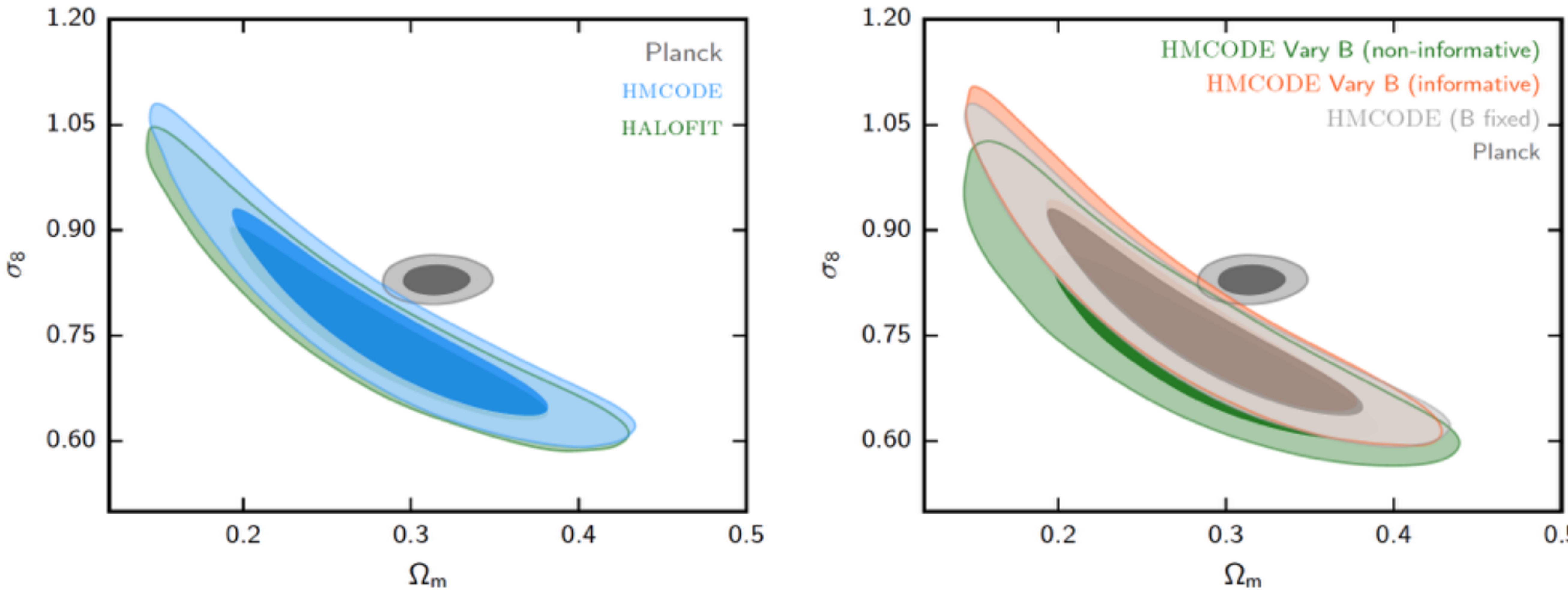


Baryonic feedback



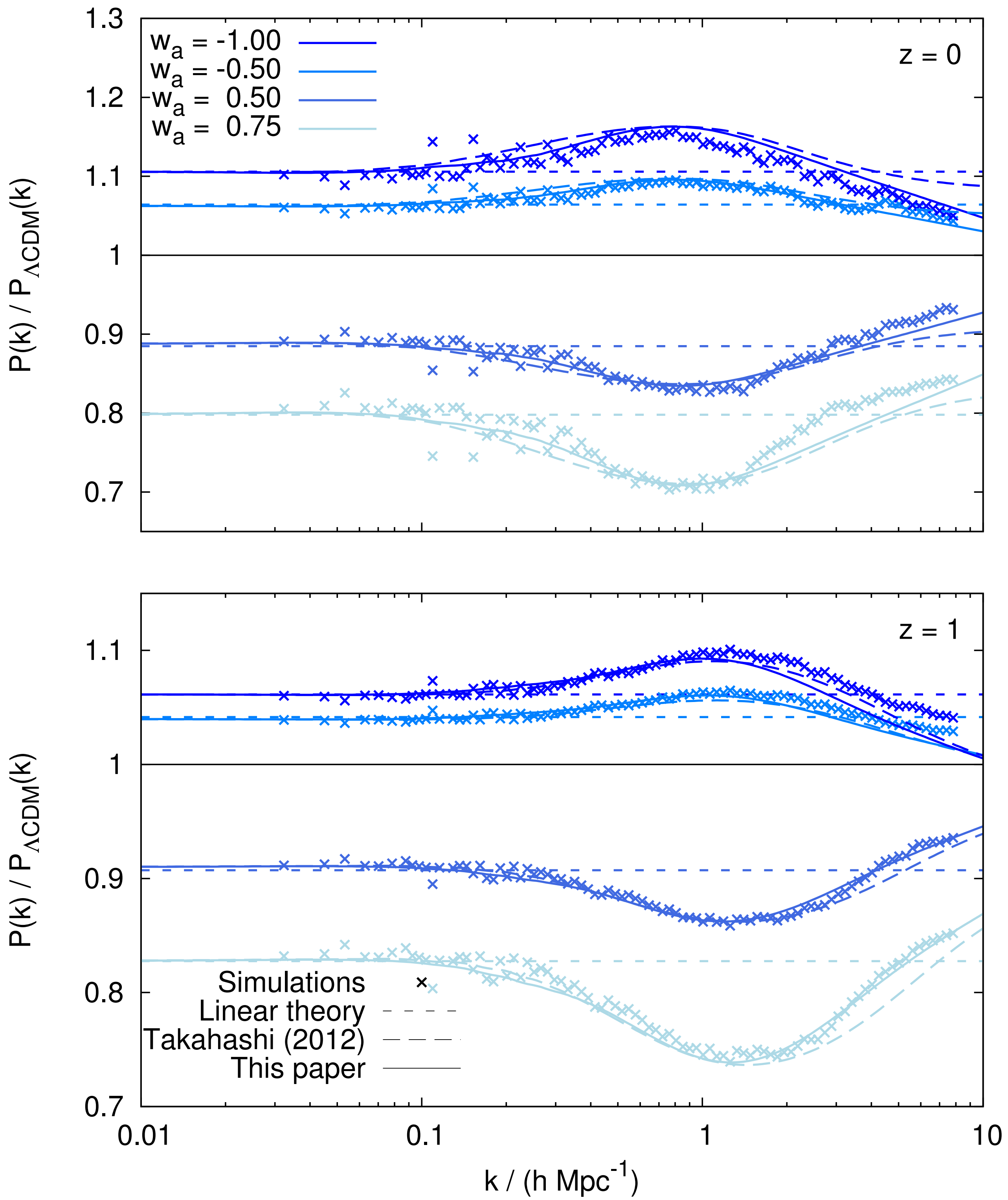


From Joudaki et al. (CFHTLenS revisited;1601.05786)

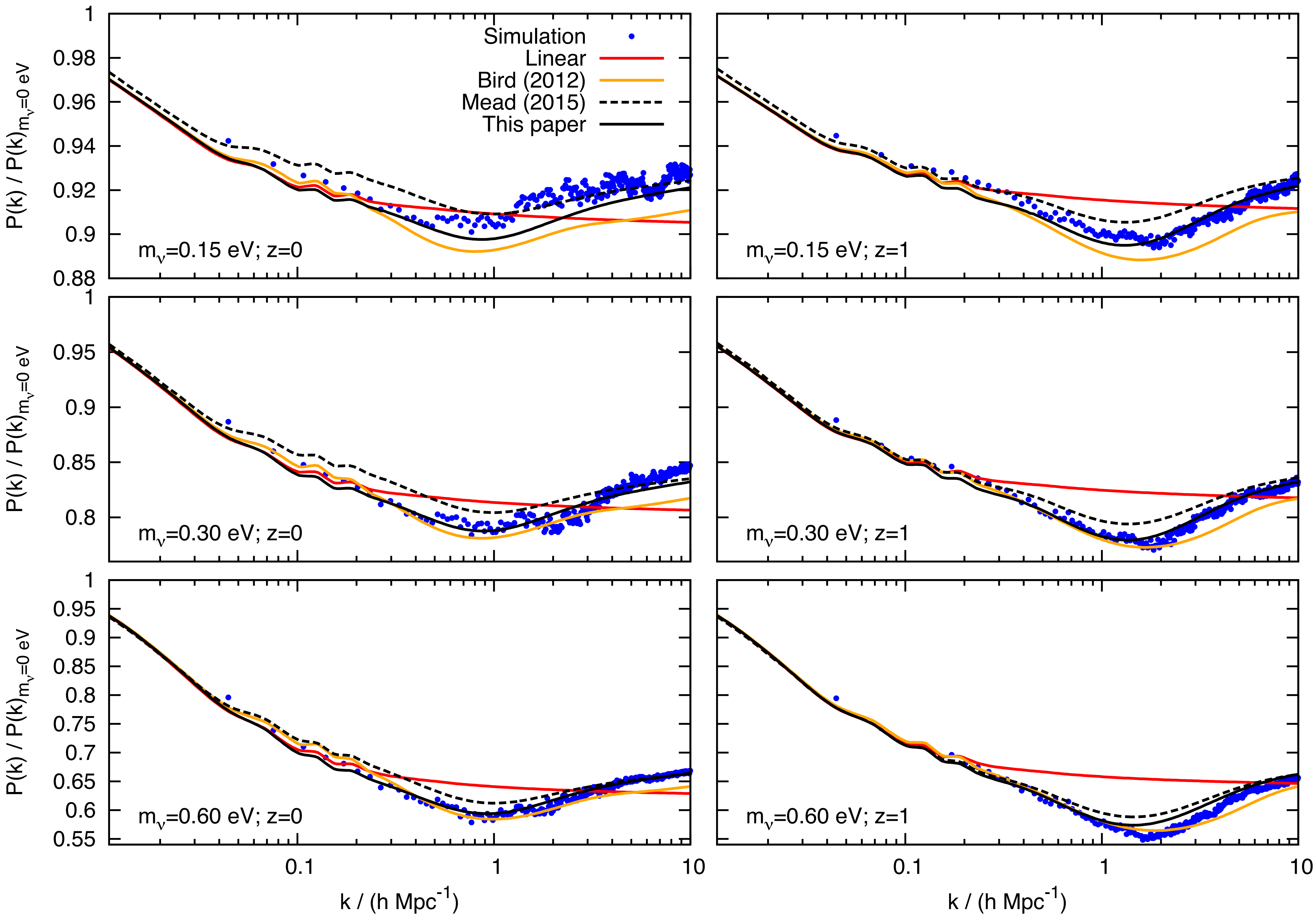


Dark energy

$$w = w_0 + (1-a)w_a$$



Massive neutrinos



Summary

- Matter power spectra accurate to 5% across a wide range of cosmological models (massive nu, dark energy)
- Publications
  - http://mnras.oxfordjournals.org/content/454/2/1958.full.pdf
  - http://mnras.oxfordjournals.org/content/459/2/1468.full.pdf
- Code available:
  - https://github.com/alexander-mead/HMcode
- Integrated into CAMB (halofit_ppf.f90 module)
- Relatively easy to add standard model extensions

Spherical collapse model

- 5% accuracy can be improved on if we work only with the power spectrum 'response'
- Consider models with fixed linear spectrum shape and amplitude, but that could differ via their growth history (e.g., dark energy)
- Look at differences in spherical-collapse model predictions.

Spherical-collapse model

$$\ddot{\delta} + 2H\dot{\delta} - \frac{4}{3}\frac{\dot{\delta}^2}{1+\delta} = 4\pi G\bar{\rho}\delta(1+\delta)$$

Spherical-collapse model

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Spherical-collapse model

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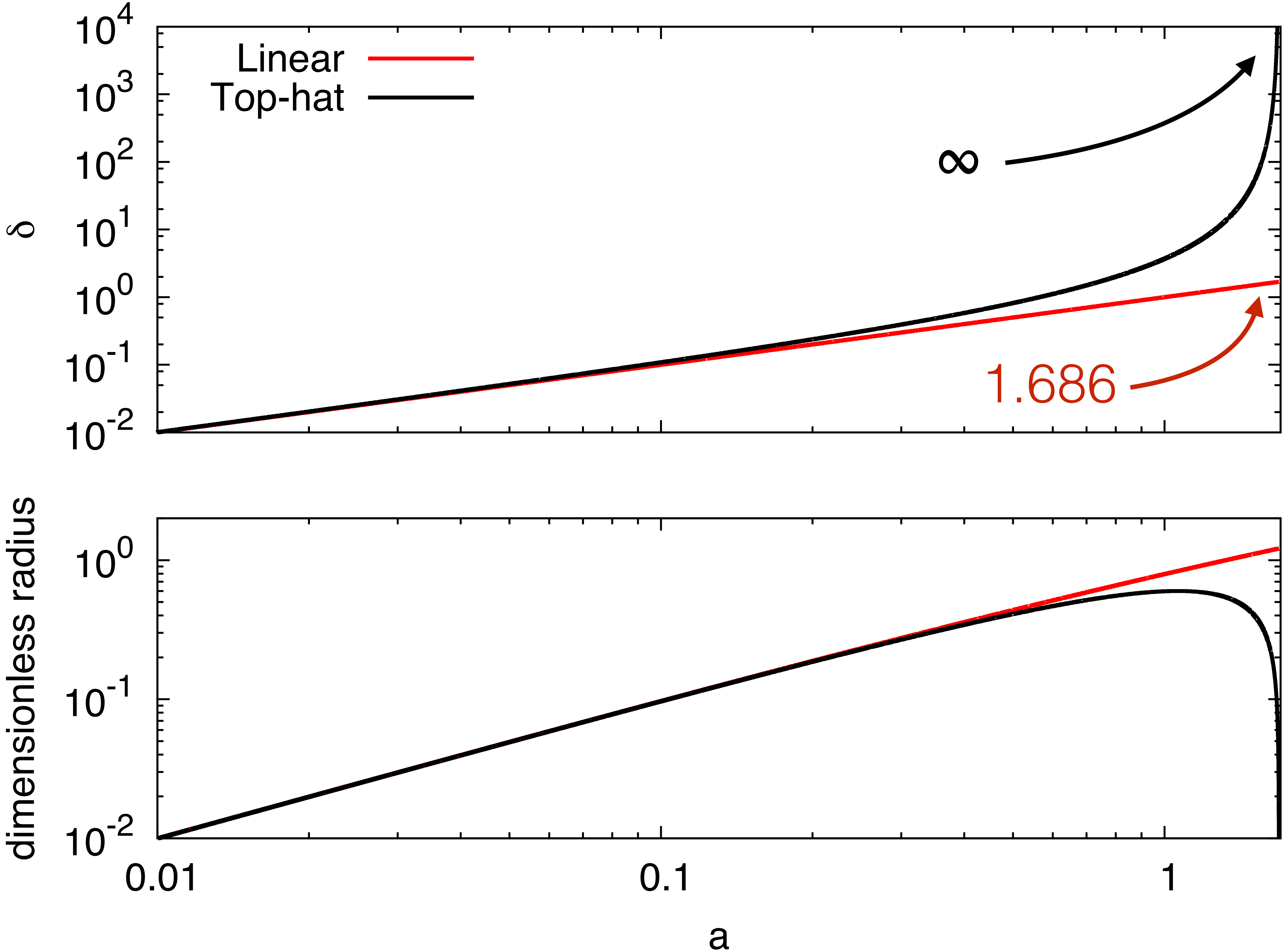
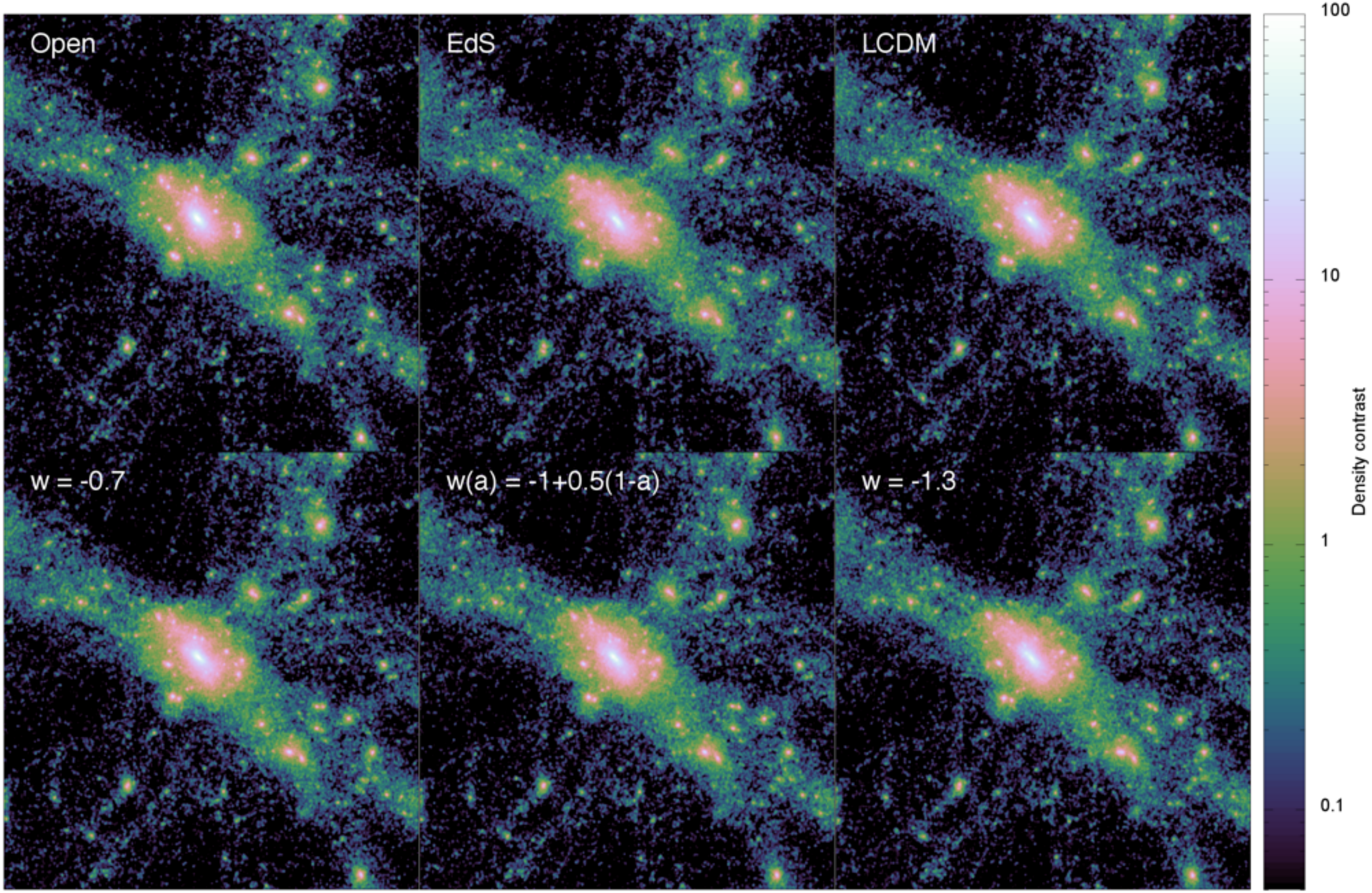
$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G\bar{\rho}\delta$$



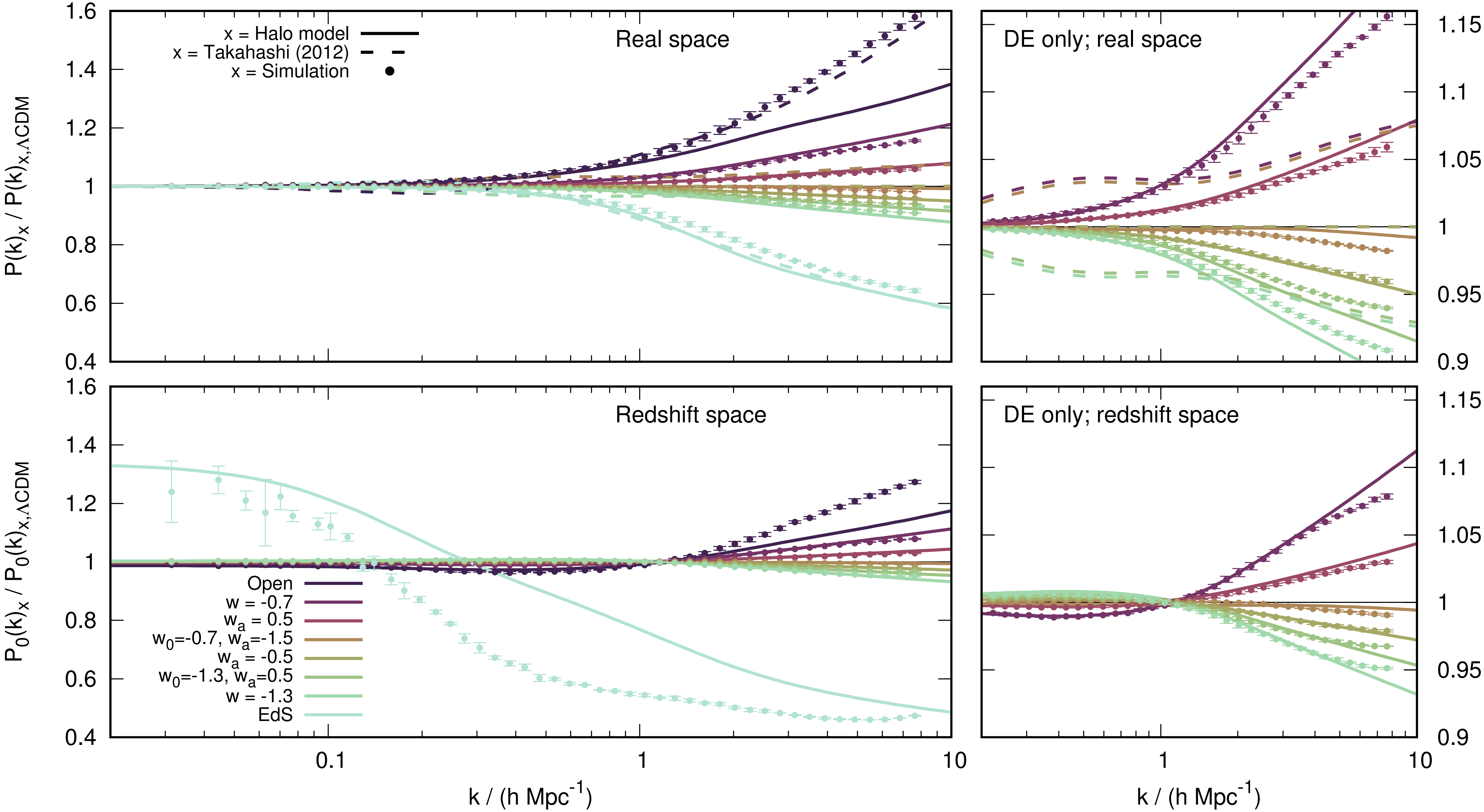
Linear
Top-hat
∞
1.686
δ
dimensionless radius
a
0.01
0.1
1
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}

Table 1. Cosmological parameters of the simulations used in this paper. Dynamical dark energy is parameterised via $w(a) = w_0 + (1 - a)w_a$ and is taken to be spatially homogeneous and thus only affects the background expansion. All simulations use 512^3 particles in cubes of size $L = 200\,h^{-1}\mathrm{Mpc}$, and start from initial conditions with identical mode phases, but with initial amplitudes adjusted to ensure $\sigma_8 = 0.8$ at $z = 0$. The shape of the linear spectrum used to generate the initial conditions is identical in each case, and was generated using CAMB (Lewis et al. 2000) with cosmological parameters $\Omega_\mathrm{m} = 0.3$, $\Omega_w = 1 - \Omega_\mathrm{m}$, $\Omega_\mathrm{b} = 0.05$, $h = 0.7$, $n_\mathrm{s} = 0.96$ and $w = -1$. For each cosmology, I ran 3 different realisations of initial conditions. Also shown are the spherical-model parameters δ_c and Δ_v from a numerical calculation.

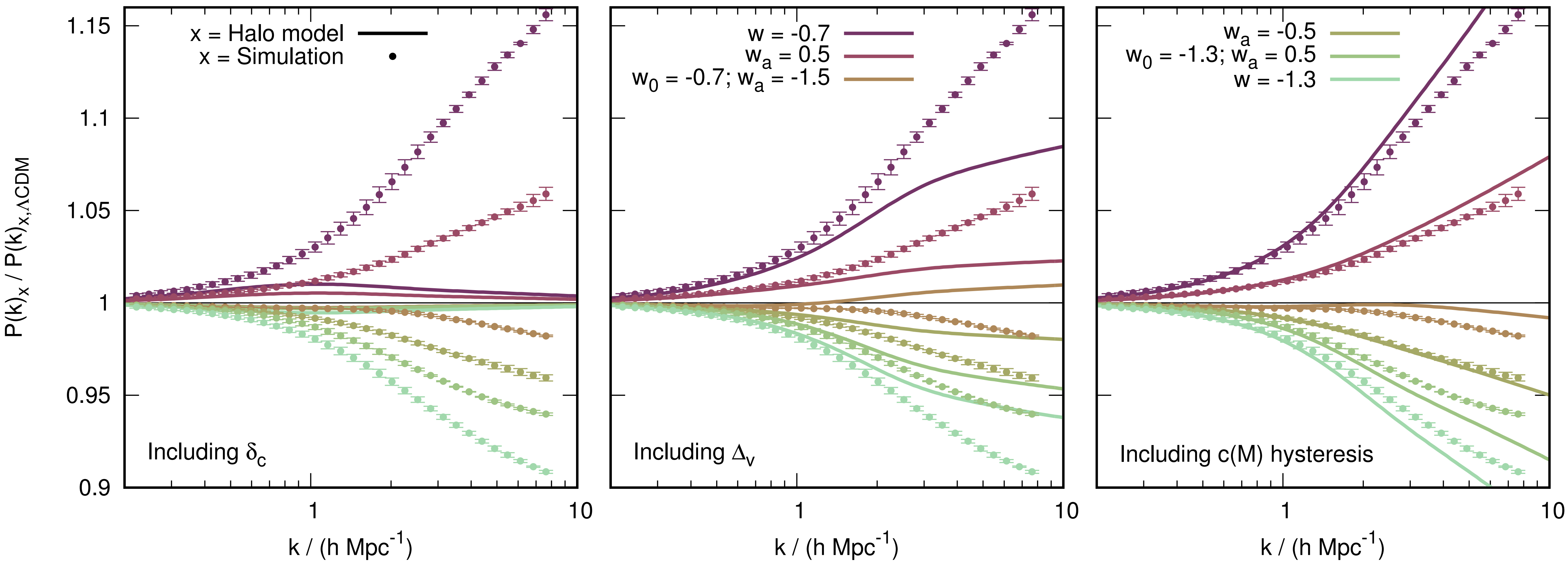
Cosmology	Ω_m	Ω_w	w_0	w_a	δ_c	Δ_v
ΛCDM	0.3	0.7	−1	0	1.6755	310.1
EdS	1.0	0.0	-	-	1.6866	177.7
Open	0.3	0.0	-	-	1.6513	402.0
DE1	0.3	0.7	−0.7	0	1.6695	342.7
DE2	0.3	0.7	−1.3	0	1.6787	282.4
DE3	0.3	0.7	−1	0.5	1.6724	318.5
DE4	0.3	0.7	−1	−0.5	1.6773	301.6
DE5	0.3	0.7	−0.7	−1.5	1.6774	313.3
DE6	0.3	0.7	−1.3	0.5	1.6771	290.1


Open
EdS
LCDM
w = -0.7
w(a) = -1+0.5(1-a)
w = -1.3
Density contrast
100
10
1
0.1

Spherical collapse model



Spherical collapse model



Summary

- Matter 'power spectrum response' accurate to 1-2% for $k < 5h\ \mathrm{Mpc}^{-1}$ (Euclid or LSST lensing accuracy)
- No fitted parameters
- Publication:
  - http://mnras.oxfordjournals.org/content/early/2016/09/13/mnras.stw2312.full.pdf
- Code available:
  - https://github.com/alexander-mead/collapse
- Papers also contain accurate fitting functions for δ_c and Δ_v for a wide range of dark energy models